1. What is the degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = \left(\frac{dy}{dx}\right)^{\frac{5}{2}} ?$$

(a) 3

(b) 2

(c) $\frac{5}{2}$

(d) $\frac{3}{2}$

2. What is $\int_n^{n+1}(x-[x])\,dx$, where $[\cdot]$ is the greatest integer function and n is natural number?

(a) $\frac{4n+1}{2}$

(b) $\frac{2n+1}{2}$

(c) $\frac{1}{2}$

(d) 1

3. Consider the following statements :

I. $y = xe^{2x}$ is the solution of $\frac{dy}{dx} = y\left(2+\frac{1}{x}\right)$.

II. $y = x\ln|x| + cx$ is the solution of $\frac{dy}{dx} = \frac{x+y}{x}$.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

4. If k is an arbitrary constant, then what is the general solution of the equation $(x+y)^2\frac{dy}{dx} = k^2$?

(a) $y+x = \tan(x+c)+k$

(b) $x+y = k\tan\left(\frac{y-c}{k}\right)$

(c) $x-y = k\tan\left(\frac{y-c}{k}\right)$

(d) $y-x = \tan(x+c)+k$

5. What is $\int\frac{dx}{10^x+10^{-x}}$ equal to?

(a) $\tan^{-1}(10^x)+c$

(b) $(\ln 10)\tan^{-1}(10^x)+c$

(c) $\frac{1}{\ln 10}\tan^{-1}(10^x)+c$

(d) $\ln(10^x+10^{-x})+c$

6. A wire of length 20 cm is to be bent into a rectangle. Which of the following statements is/are correct?

I. The rectangle of the largest area is the square.

II. It is possible to form a rectangle of an area of 27 cm^2.

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

7. If $I_1 = \int_e^{e^2} \frac{dx}{\ln x}$ and $I_2 = \int_1^2 \frac{e^x}{x}\,dx$, then which one of the following is correct?

(a) $I_1 - I_2 = 0$

(b) $I_1 + I_2 = 0$

(c) $I_1 - 2I_2 = 0$

(d) $2I_1 - I_2 = 0$

8. What is the area of the region bounded by $|x| \leq 2k$ and $|y| \leq k$, where k is a positive real number?

(a) $2k^2$

(b) $4k^2$

(c) $5k^2$

(d) $8k^2$

9. Consider the following statements regarding the function $f(x) = \frac{1}{x-5}$:

Statement–I :

$f(x)$ is decreasing on the intervals $x < 5$ and $x > 5$.

Statement–II :

$f'(x) > 0$ for all $x \neq 5$.

Which one of the following is correct in respect of the above statements?

(a) Both Statement–I and Statement–II are correct and Statement–II explains Statement–I

(b) Both Statement–I and Statement–II are correct but Statement–II does not explain Statement–I

(c) Statement–I is correct but Statement–II is not correct

(d) Statement–I is not correct but Statement–II is correct

10. Consider the following statements :

Statement–I :

The function $f(x) = \frac{x^3 + 128}{x}$ has a minimum value 48 at $x = 4$.

Statement–II :

As x increases through 4, $f'(x)$ changes sign from positive to negative.

Which one of the following is correct in respect of the above statements?

(a) Both Statement–I and Statement–II are correct and Statement–II explains Statement–I

(b) Both Statement–I and Statement–II are correct but Statement–II does not explain Statement–I

(c) Statement–I is correct but Statement–II is not correct

(d) Statement–I is not correct but Statement–II is correct

For the following **two (02)** items :

Let

$$f(x) = \begin{cases} \frac{1-\cos 2x}{x^2}, & x < 0 \\ 9, & x = 0 \\ \frac{\sqrt{x}}{\sqrt{(16+\sqrt{x})} - 4}, & x > 0 \end{cases}$$

11. What is $\lim_{x \to 0-} f(x)$ equal to?

(a) 2

(b) 4

(c) 6

(d) 8

12. What is $\lim_{x \to 0+} f(x)$ equal to?

(a) 6

(b) 7

(c) 8

(d) 9

For the following **three (03)** items :

Consider the function $f(x) = x|x|$.

13. What is $\lim_{x \to -1} f(x)$ equal to?

(a) -1

(b) 0

(c) 1

(d) Limit does not exist

14. What is the area bounded by the curve $f(x)$, the x-axis and the lines $x = -2$ and $x = 1$?

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) $\frac{5}{2}$

(d) 3

15. Consider the following statements :

I. The function is increasing in the interval $(-\infty, \infty)$.

II. The function is differentiable at $x = 0$.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

For the following **two (02)** items :

Consider the function

$$f(x) = \frac{x}{1-x} \quad (x > 0, x \neq 1)$$

16. What is $\frac{f(x)}{f(x+1)}$ equal to?

(a) $-f(x^2)$

(b) $-f(\sqrt{x})$

(c) $f(x^2)$

(d) $f(x-1)$

17. What is $(1-x)f(\sqrt{x})+xf(\sqrt{x}+1)$ equal to?

(a) $-f(x)$

(b) $f(x)$

(c) x

(d) 0

For the following **three (03)** items :

Let $y=f(x)=\dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}+\ln\sqrt{1-x^2}$.

18. What is the slope of the tangent to the curve $y=f(x)$ at $x=0\cdot5$?

(a) $4\pi\sqrt{3}/27$

(b) $8\pi\sqrt{3}/27$

(c) 4π

(d) 8π

19. What is $\dfrac{d^2y}{dx^2}$ at $x=0$ equal to?

(a) 0

(b) 0·5

(c) 1

(d) 1·5

20. If $x=\sin\theta$, then what is $\dfrac{dy}{dx}$ equal to?

(a) $\theta\sec\theta$

(b) $\theta\sec^2\theta$

(c) $\theta\sec^3\theta$

(d) $2\tan\theta+\theta\sec^2\theta$

For the following **two (02)** items :

A plane P is parallel to the line having direction ratios $\langle 1, 3, 2\rangle$ and contains the line of intersection of the planes $6x+4y-5z=2$ and $x-2y+3z=0$.

21. Which of the following are the direction ratios of the line of intersection of the given planes?

(a) $\langle 2, 23, 16\rangle$

(b) $\langle 2, -23, -16\rangle$

(c) $\langle 2, 3, 2\rangle$

(d) $\langle -1, 3, -2\rangle$

22. What is the equation of the plane P?

(a) $2x-20y+29z+2=0$

(b) $2x-20y+29z-2=0$

(c) $2x+3y+2z-4=0$

(d) $x-3y+2z+5=0$

For the following **two (02)** items :

Suppose S is the sphere with the smallest radius that passes through the points $A(1, 0, 0)$, $B(0, 1, 0)$ and $C(0, 0, 1)$.

23. What is the radius of S?

(a) $\sqrt{\frac{1}{3}}$

(b) $\sqrt{\frac{2}{3}}$

(c) $\frac{1}{3}$

(d) 1

24. On which one of the following planes does the centre of S lie?

(a) $x+y+z-1=0$

(b) $x+y+z+1=0$

(c) $3x+3y+3z-1=0$

(d) $3x+3y+3z+1=0$

For the following **two (02)** items :

Let $A(1, -1, 0)$, $B(-2, 1, 8)$ and $C(-1, 2, 7)$ are three consecutive vertices of a parallelogram $ABCD$.

25. What is the fourth vertex D?

(a) $(0, -2, 1)$

(b) $(2, 0, -1)$

(c) $(1, 0, 1)$

(d) $(1, 2, 0)$

26. If angle BCD is θ, then what is $\cos^2\theta$ equal to?

(a) 26/77

(b) 27/77

(c) 82/237

(d) 83/237

27. For different values of m, the equation $4y = mx - m + 2$ represents

(a) parallel lines

(b) concurrent lines

(c) lines at a fixed distance from the origin of coordinates

(d) the same line

28. The equation of the locus of a point equidistant from the points (a, b) and (c, d) is $(a-c)x+(b-d)y+k=0$. What is the value of k?

(a) $a^2-c^2+b^2-d^2$

(b) $c^2+d^2-a^2-b^2$

(c) $(a^2-c^2+b^2-d^2)/2$

(d) $(c^2+d^2-a^2-b^2)/2$

29. Consider the following statements in respect of the equation $x^2 + 3y = 0$:

I. The equation represents the equation to parabola that opens upwards.

II. The axis of the parabola is $x = 0$.

III. The equation of the latus rectum is $4y - 3 = 0$.

How many of the statements given above are correct?

(a) None

(b) One

(c) Two

(d) All three

30. What is the sum of the intercepts of the line

$$\frac{x}{a^2} + \frac{y}{b^2} = \frac{2}{a^2 + b^2}$$

on the coordinate axes?

(a) 2

(b) 1

(c) $\frac{1}{2}$

(d) $a^2 + b^2$

31. What is the harmonic mean of the numbers $C(10, 3)$, $C(10, 4)$, $C(10, 5)$, $C(10, 6)$ and $C(10, 7)$?

(a) 3150/19

(b) 4000/19

(c) 252

(d) 225

32. In a sample survey of a village, the probability that a farmer is in debt is 0·60. What is the probability that three randomly selected farmers are all in debt (assume independence of events)?

(a) 0·000216

(b) 0·064

(c) 0·216

(d) 0·512

33. The probability that a family owns a laptop is 0·68; that it also owns a desktop is 0·56. If the probability that it owns both is 0·48, then what is the probability that a randomly selected family owns a laptop or a desktop?

(a) 0·80

(b) 0·76

(c) 0·36

(d) 0·28

34. An urn contains 10 white and 5 red balls. If two balls are drawn at random, then what is the probability that both the balls are red?

(a) 2/21

(b) 1/7

(c) 4/21

(d) 3/7

35. An urn contains 5 white, 6 red and 4 blue balls. Three balls are drawn at random. What is the probability that a white ball, a red ball and a blue ball are drawn?

(a) 28/91

(b) 2/7

(c) 24/91

(d) 23/91

36. Under which of the following conditions may binomial distribution be used?

I. The number of trials is infinite and not fixed.

II. The trials are independent.

III. Each trial has two possible outcomes.

Select the correct answer using the code given below.

(a) II only

(b) III only

(c) I and II

(d) II and III

37. A person X speaks the truth 4 out of 5 times and person Y speaks the truth 5 out of 6 times. What is the probability that they will contradict each other in stating the fact?

(a) 3/10

(b) 1/15

(c) 1/6

(d) 7/10

38. The probability that a student passes Physics test is 2/3 and the probability that he passes both Physics test and English test is 11/15. The probability that he passes at least one test is 4/5. What is the probability that he passes English test?

(a) 11/15

(b) 13/15

(c) 14/15

(d) 1

39. An event X can happen with probability p and event Y can happen with probability q. Further, X and Y are independent events. Which of the following statements is/are correct?

I. The probability that exactly one of the events happens is $p+q-pq$.

II. The probability that at least one of the events happens is $p+q-2pq$.

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

40. Three faces of a die are black, two faces are white and one face is red. The die is tossed three times. What is the probability that the colours black, white and red appear in the first, second and third tosses respectively?

(a) 1/36

(b) 1/6

(c) 7/36

(d) 5/36

41. A fair coin is tossed 4 times. What is the probability that two heads do not occur consecutively?

(a) 1/8

(b) 3/8

(c) 7/16

(d) 1/2

42. In a throw of three dice, what is the probability of getting one prime number, one composite number and one number which is neither prime nor composite?

(a) 1/2

(b) 1/3

(c) 1/4

(d) 1/6

43. An integer is chosen at random from the first 50 integers. What is the probability that the integer is neither divisible by 5 nor 9?

(a) 7/10

(b) 18/25

(c) 37/50

(d) 19/25

44. Out of 50 consecutive natural numbers, two integers are chosen at random. What is the probability that their sum is odd?

(a) 1/2

(b) 24/49

(c) 1/4

(d) 25/49

45. The standard deviation of 100 observations is 10. If 20 is added to each observation, then what will be the new standard deviation?

(a) 10

(b) 15

(c) 20

(d) 25

46. Let X be a random variable following binomial distribution with parameters $n = 5$ and $p = k$. Further, $P(X = 1) = 0{\cdot}4096$ and $P(X = 2) = 0{\cdot}2048$. What is the value of k?

(a) 0·2

(b) 0·25

(c) 0·3

(d) 0·35

47. The frequency distribution of the marks obtained by students in a Science examination is given below :

Marks	5–15	15–25	25–35	35–45
Number of students	20	30	30	20

What is the arithmetic mean?

(a) 20

(b) 25

(c) 30

(d) 35

48. If $P(A) = 0{\cdot}3$, $P(B) = 0{\cdot}4$ and $P(A|B) = 0{\cdot}5$, then what is the value of $P(B|A)$?

(a) 0·325

(b) 0·333

(c) 0·375

(d) 0·667

49. If $P(A) = 1/3$, $P(B) = 1/2$ and $P(A \cap B) = 1/4$, then what is the value of $P(\bar{A} \cup B)$?

(a) 7/12

(b) 2/3

(c) 3/4

(d) 11/12

50. Consider the following statements :

I. Mean and variance have the same unit of measurement.

II. Mean deviation and standard deviation have the same unit of measurement.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

For the following **two (02)** items :

Let α and β be the roots of the quadratic equation

$$x^2 + (\log_{0.5}(a^2))x + (\log_{0.5}(a^2))^4 = 0$$

where $a^2 \neq 1$ and $\log_{0.5}(a^2) > 0$. Further, $\beta^2 = \alpha(\log_{a^2}(0 \cdot 5))$.

51. What is β equal to?

(a) $\log_{a^2}(0 \cdot 5)$

(b) $\log_{0.5}(a^2)$

(c) $2(\log_{a^2}(0 \cdot 5))$

(d) $2\log_{0.5}(a^2)$

52. What is the relation between α and β?

(a) $\alpha = 2\beta$

(b) $2\alpha = \beta$

(c) $\alpha = -2\beta$

(d) $2\alpha = -\beta$

For the following **two (02)** items :

Let $p = \sum_{j=1}^{n} \log_{10} 2^j$ and $q = \sum_{j=1}^{n} \log_{10} 5^j$.

53. If $p+q = 66$, then which one of the following is correct?

(a) $n < 7$

(b) $7 < n < 9$

(c) $9 < n < 12$

(d) $n > 12$

54. If $p+q = 15$, then what is $q-p$ equal to?

(a) $\log_{10} 2 \cdot 5$

(b) $5\log_{10} 2 \cdot 5$

(c) $10\log_{10} 2 \cdot 5$

(d) $15\log_{10} 2 \cdot 5$

For the following **two (02)** items :

Let $\sin A + \sin B = p$ and $\cos A + \cos B = q$.

55. What is $\frac{p}{q}$ equal to?

(a) $\tan\left(\frac{A-B}{2}\right)$

(b) $\cot\left(\frac{A-B}{2}\right)$

(c) $\tan\left(\frac{A+B}{2}\right)$

(d) $\cot\left(\frac{A+B}{2}\right)$

56. What is $\frac{p^2-q^2}{p^2+q^2}$ equal to?

(a) $\cos(A+B)$

(b) $\cos(A-B)$

(c) $\cos\left(\frac{\pi}{2}-A-B\right)$

(d) $\cos(\pi-A-B)$

For the following **two (02)** items :

Let $p = \text{cosec } 20°$ and $q = \text{cosec } 70°$.

57. What is $\left(\frac{\sqrt{3}p}{4} - \frac{q}{4}\right)$ equal to?

(a) -1

(b) 0

(c) 1

(d) 2

58. What is $\frac{p^2+q^2}{p^2q^2}$ equal to?

(a) $\frac{1}{2}$

(b) 1

(c) $\frac{3}{2}$

(d) 2

For the following **two (02)** items :

Let $\cos(2x+3y)=\frac{1}{2}$ and $\cos(3x+2y)=\frac{\sqrt{3}}{2}$, where $-\pi<(2x+3y)<\pi$ and $-\pi<(3x+2y)<\pi$.

59. How many values does $(x+y)$ have?

(a) Two

(b) Three

(c) Four

(d) More than four

60. How many values does $(y-x)$ have?

(a) Two

(b) Three

(c) Four

(d) More than four

61. In a class of 45 students, 34 like to play cricket and 26 like to play football. Further, each student likes to play at least one of the two games. How many students like to play exactly one game?

(a) 45

(b) 30

(c) 25

(d) 15

62. The system of equations

$$2x-3y-5=0,\ 15y-10x+50=0$$

(a) has a unique solution

(b) has infinitely many solutions

(c) is inconsistent

(d) is consistent and has exactly two solutions

63. If

$$\left(\frac{1-i}{1+i}\right)^{2m}\left(\frac{1+i}{1-i}\right)^{2n}=1$$

where $i=\sqrt{-1}$, then what is the smallest positive value of $(m-n)$?

(a) 1

(b) 2

(c) 4

(d) 8

64. In obtaining the solution of the system of equations $x+y+z=7$, $x+2y+3z=16$ and $x+3y+4z=22$ by Cramer's rule, the value of y is obtained by dividing D by D_2, where

$$D=\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & 4\end{vmatrix}$$

What is the value of the determinant D_2?

(a) -13

(b) -3

(c) 3

(d) 13

65. Consider the following in respect of non-singular matrices A and B :

I. $(AB)^{-1}=A^{-1}B^{-1}$

II. $(BA)(AB)^{-1}=I$, where I is the identity matrix

III. $(AB)^{T}=A^{T}B^{T}$

How many of the above are correct?

(a) None

(b) One

(c) Two

(d) All three

66. The value of the determinant

$$\begin{vmatrix}a & b & c\\ l & m & n\\ p & q & r\end{vmatrix}$$

is equal to

(a) $\begin{vmatrix}a & b & c\\ p & q & r\\ l & m & n\end{vmatrix}$

(b) $\begin{vmatrix}l & m & n\\ a & b & c\\ p & q & r\end{vmatrix}$

(c) $\begin{vmatrix}p & q & r\\ a & b & c\\ l & m & n\end{vmatrix}$

(d) $\begin{vmatrix}a & p & l\\ b & q & m\\ c & r & n\end{vmatrix}$

67. Let 1, ω, ω^2 be three cube roots of unity. If $x=a+b$, $y=a\omega+b\omega^2$, $z=a\omega^2+b\omega$, then what is $x^2+y^2+z^2$ equal to?

(a) $6ab$

(b) $3ab$

(c) a^2+b^2

(d) 1

68. How many 4-digit numbers that are divisible by 4 can be formed using the digits 1, 2, 3 and 4 (repetition of digits is not allowed)?

(a) 3

(b) 6

(c) 9

(d) 12

69. If a, b, c are the sides of a triangle ABC and p is the perimeter of the triangle, then what is

$$\begin{vmatrix} p+c & a & b \\ c & p+a & b \\ c & a & p+b \end{vmatrix}$$

equal to?

(a) p^3

(b) $2p^3$

(c) $3p^3$

(d) $4p^3$

70. Which one of the following is the greatest coefficient in the expansion of $(1+x)^{100}$?

(a) The coefficient of x^{100}

(b) The coefficient of x^{99}

(c) The coefficient of x^{51}

(d) The coefficient of x^{50}

71. If $p^x = q^y = r^z$, where x, y and z are in GP, then consider the following statements :

I. p, q and r are in AP.

II. $\ln p$, $\ln q$ and $\ln r$ are in GP.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

72. If A and B are non-empty subsets of a set, and A^c and B^c represent their complements, then which of the following is/are correct?

I. $A - B = B^c - A^c$

II. $A - B^c = A^c - B$

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

73. Let $y = x!$ and $z = (2x)!$. If $(z/y) = 120$, then what is the value of $(3x)!$?

(a) 362880

(b) 181440

(c) 90720

(d) 45360

74. Let n be a natural number. The number of consecutive zeros at the end of the expansion of $n!$ is exactly 2. How many values of n are possible?

(a) 3

(b) 4

(c) 5

(d) More than 5

75. If $(10+\log_{10} x)$, $(10+\log_{10} y)$ and $(10+\log_{10} z)$ are in AP, then consider the following statements :

I. The GM of x and z is y^2.

II. The AM of $\log_{10} x$ and $\log_{10} z$ is $\log_{10} y$.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

76. How many terms of the series $1+3+5+7+\ldots$ amount to a sum equal to 12345678987654321?

(a) 11111111

(b) 110000011

(c) 111101111

(d) 111111111

77. How many terms are identical in the two APs 19, 21, 23, ... up to 110 terms and 19, 22, 25, 28, ... up to 75 terms?

(a) 35

(b) 36

(c) 37

(d) 38

78. If

$$\alpha = \frac{-1+\sqrt{-3}}{2}$$

then what is the value of

$(1+\alpha^{19}-\alpha^{35})^{100}-(1-3\alpha^{25}+\alpha^{38})^{50}$?

(a) –2

(b) –1

(c) 0

(d) 2

79. What is the remainder when 5^{99} is divided by 13?

(a) 10

(b) 9

(c) 8

(d) 6

80. What is the value of the determinant of the inverse of the matrix

$$\begin{bmatrix} -4 & -5 \\ 2 & 2 \end{bmatrix}?$$

(a) $\frac{1}{2}$

(b) 1

(c) 2

(d) 4

81. Which one of the following is the perpendicular form of the straight line $\sqrt{3}x + 2y = 7$?

(a) $y = -\frac{\sqrt{3}}{2}x + \frac{7}{2}$

(b) $\frac{x}{\left(\frac{7}{\sqrt{3}}\right)} + \frac{y}{\left(\frac{7}{2}\right)} = 1$

(c) $\frac{\sqrt{3}}{\sqrt{7}}x + \frac{2}{\sqrt{7}}y = \sqrt{7}$

(d) $\frac{\sqrt{3}}{\sqrt{7}}x + \frac{2}{\sqrt{7}}y = 7$

82. If the vertices B and D of a square $ABCD$ are (2, 3) and (4, 1) respectively, then what is the area of the square?

(a) 2 square units

(b) 3 square units

(c) 4 square units

(d) 8 square units

83. What is the value of $\sin\theta$ if θ is the acute angle between the lines whose equations are $px + qy = p + q$ and $p(x - y) + q(x + y) = 2q$?

(a) $\frac{\sqrt{3}}{2}$

(b) $\frac{3}{4}$

(c) $\frac{1}{2}$

(d) $\frac{1}{\sqrt{2}}$

84. The circle $x^2 + y^2 - 2kx - 2ky + k^2 = 0$ touches the x-axis at P and y-axis at Q. What is PQ equal to?

(a) $\sqrt{2}k$

(b) $2k$

(c) $2\sqrt{2}k$

(d) $4k$

85. What is the distance between the foci of the hyperbola $x^2 - 4y^2 = 1$?

(a) $\sqrt{3}$

(b) $\sqrt{5}$

(c) $2\sqrt{3}$

(d) $2\sqrt{5}$

86. Let $\vec{p} = \vec{a} - \vec{b}$, $\vec{q} = \vec{a} + \vec{b}$. If $|\vec{a}| = |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 2$, then what is the value of $|\vec{p} \times \vec{q}|$?

(a) $\sqrt{3}$

(b) $\sqrt{6}$

(c) $2\sqrt{3}$

(d) $4\sqrt{3}$

87. How many of the following can be a vector perpendicular to both the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + 3\hat{k}$?

I. $4\hat{i} + 5\hat{j} - 3\hat{k}$

II. $-8\hat{i} - 10\hat{j} + 6\hat{k}$

III. $\frac{1}{50}(-4\hat{i} - 5\hat{j} + 3\hat{k})$

Select the correct answer.

(a) None

(b) One

(c) Two

(d) All three

88. What is the area of the parallelogram whose sides are represented by the vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} + 2\hat{k}$?

(a) $\frac{1}{2}\sqrt{26}$ square units

(b) $\frac{1}{2}\sqrt{27}$ square units

(c) $\sqrt{26}$ square units

(d) $\sqrt{27}$ square units

89. The position vectors of the vertices A, B, C and D of a quadrilateral $ABCD$ are given by $3\hat{i} + 4\hat{j} - 2\hat{k}$, $4\hat{i} - 4\hat{j} - 3\hat{k}$, $2\hat{i} - 3\hat{j} + 2\hat{k}$ and $6\hat{i} - 2\hat{j} + \hat{k}$ respectively. What is the angle between the diagonals AC and BD of the quadrilateral?

(a) 90°

(b) 75°

(c) 60°

(d) 45°

90. A force $\vec{F} = 2\hat{i} - \lambda\hat{j} + 5\hat{k}$ is applied at the point $A(1, 2, 5)$. If its moment about the point $B(-1, -2, 3)$ is $16\hat{i} - 6\hat{j} + 2\lambda\hat{k}$, then what is the value of λ?

(a) -2

(b) 0

(c) 1

(d) 2

For the following **two (02)** items :

Consider the function $f(x) = 1 - \sqrt[3]{(x-1)^2}$.

91. What is the domain of the function?

(a) $(1, \infty)$

(b) $(-\infty, \infty)$

(c) $(0, \infty)$

(d) $(-\infty, \infty) \setminus \{1\}$

92. The function has

(a) a minimum at $x = 1$

(b) a maximum at $x = 1$

(c) neither maximum nor minimum at $x = 1$

(d) no extremum

For the following **two (02)** items :

Consider the function

$$f(x) = \begin{cases} 4(5^x), & x < 0 \\ 8k + x, & x \geq 0 \end{cases}$$

93. If the function is continuous, then what is the value of k?

(a) 0·5

(b) 1

(c) 1·5

(d) 2

94. What is $f'(-1)$ equal to?

(a) $\frac{2}{5}\ln 5$

(b) $\frac{3}{5}\ln 5$

(c) $\frac{4}{5}\ln 5$

(d) $20\ln 5$

For the following **two (02)** items :

Let $u = \int e^x \cos x \, dx$ and $v = \int e^x \sin x \, dx$.

95. What is $u + v$ equal to?

(a) $-\frac{du}{dx}$

(b) $-\frac{dv}{dx}$

(c) $\frac{du}{dx}$

(d) $\frac{dv}{dx}$

96. Consider the following :

I. $\frac{du}{dx} = -v$

II. $\frac{dv}{dx} = -u$

Which of the above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

For the following **two (02)** items :

Let the function $f(x) = |x-3| + |x-4|$ be defined on the interval $[0, 5]$.

97. What is $\frac{dy}{dx}$ at $x = 3 \cdot 5$ equal to?

(a) 0

(b) 1

(c) 2

(d) 3·5

98. Consider the following statements :

I. The function is differentiable at $x = 3$.

II. The function is differentiable at $x = 4$.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

For the following **two (02)** items :

Consider the function $f(x) = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$.

99. What is $f \circ f \circ f \circ f \circ f(0)$ equal to?

(a) 0

(b) 1

(c) 5

(d) 10

100. What is the inverse of the function?

(a) $\log_{10}(2x-1)$

(b) $\frac{1}{2}\log_{10}(2x-1)$

(c) $\frac{1}{4}\log_{10}\left(\frac{2x}{2-x}\right)$

(d) $\frac{1}{2}\log_{10}\left(\frac{1+x}{1-x}\right)$

101. What is the sum of the binary numbers $(101101101)_2$ and $(100011)_2$?

(a) $(110010000)_2$

(b) $(110001000)_2$

(c) $(110000100)_2$

(d) $(100100000)_2$

102. Set X contains $3n$ elements and set Y contains $2n$ elements, and they have n elements in common. How many elements does $(X-Y)\times(Y-X)$ have?

(a) $5n^2$

(b) $4n^2$

(c) $3n^2$

(d) $2n^2$

103. Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$ and $B = \{0, 1, 4, 9\}$. How many elements does the subset of $A \times B$ corresponding to the relation $R = \{(x, y) : |x| < y\}$ have, where $x \in A$ and $y \in B$?

(a) 9

(b) 12

(c) 15

(d) 16

104. Consider the following statements :

Statement–I :

If X is an $n \times n$ matrix, then $\det(mX) = m^n \det(X)$, where m is a scalar.

Statement–II :

If Y is a matrix obtained from X by multiplying any row or column by a scalar m, then $\det(Y) = m \det(X)$.

Which one of the following is correct in respect of the above statements?

(a) Both Statement–I and Statement–II are correct and Statement–II explains Statement–I

(b) Both Statement–I and Statement–II are correct but Statement–II does not explain Statement–I

(c) Statement–I is correct but Statement–II is not correct

(d) Statement–I is not correct but Statement–II is correct

105. Consider the following statements about the matrix $M = \begin{bmatrix} 71 & 23 & 48 \\ 57 & 28 & 29 \\ 65 & 17 & 48 \end{bmatrix}$:

Statement–I : The inverse of M does not exist.

Statement–II : M is non-singular.

Which one of the following is correct in respect of the above statements?

(a) Both Statement–I and Statement–II are correct and Statement–II explains Statement–I

(b) Both Statement–I and Statement–II are correct but Statement–II does not explain Statement–I

(c) Statement–I is correct but Statement–II is not correct

(d) Statement–I is not correct but Statement–II is correct

106. What is

$$\cot^{-1} 9 + \operatorname{cosec}^{-1}\left(\frac{\sqrt{41}}{4}\right)$$

equal to?

(a) $\frac{\pi}{4}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$

(d) π

114. How many values of m are possible?

(a) None

(b) One

(c) Two

(d) More than two

For the following **two (02)** items :

There are 8 points on a plane out of which 4 points are collinear.

115. How many triangles can be formed by joining these points?

(a) 56

(b) 54

(c) 53

(d) 52

116. How many quadrilaterals can be formed by joining these points?

(a) 70

(b) 69

(c) 53

(d) None of the above

For the following **two (02)** items :

Let $f(x) = ax^2 + bx + c$ be a quadratic polynomial such that $f(1) = f(4) = 2$. Further, 2 is a root of $f(x) = 0$.

117. What is the other root of $f(x) = 0$?

(a) 1

(b) 2

(c) 3

(d) Cannot be determined

118. What is $(a + b + c)$ equal to?

(a) 0

(b) 1

(c) 2

(d) Cannot be determined

For the following **two (02)** items :

Let

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

119. What is the value of the determinant of the matrix A^4?

(a) 0

(b) 1

(c) $\cos 4\theta - \sin 4\theta$

(d) $\cos^2 4\theta - \sin^2 4\theta$

120. What is $[\text{adj } A]^{-1}$ equal to?

(a) $-A$

(b) $-A^T$

(c) A

(d) A^T