For the following **two (02)** items that follow :

From the top of a building, the angles of depression of the top and bottom of a tower of height h are α and β respectively. Let H be the height of the building and D be the horizontal distance between the building and the tower.

1. What is $H^2 + D^2$ equal to?

(a) $\dfrac{h^2 \cos^2 \alpha}{\sin^2(\beta - \alpha)}$

(b) $\dfrac{h^2 \cos^2 \beta}{\sin^2(\beta - \alpha)}$

(c) $\dfrac{h^2 \cos^2 \alpha}{\sin^2(\beta + \alpha)}$

(d) $\dfrac{h^2 \cos^2 \beta}{\sin^2(\beta + \alpha)}$

2. If $\alpha = 15°$ and $\beta = 45°$, then which one of the following is correct?

(a) $H = \sqrt{2}D$

(b) $D = \sqrt{2}H$

(c) $2H = \sqrt{3}D$

(d) $H = D$

For the next **two (02)** items that follow :

The angles A, B and C of a triangle ABC are in the ratio $1 : 2 : 7$.

3. What is the ratio of the side of greatest length to the side of least length?

(a) $7 : 1$

(b) $\sqrt{7} : 1$

(c) $(3 + \sqrt{5}) : 2$

(d) $(3 + \sqrt{5}) : 3$

4. What is $\sin A \cdot \cos B$ equal to?

(a) $1/4$

(b) $1/2$

(c) 1

(d) 2

For the next **two (02)** items that follow :

In a triangle ABC, $\dfrac{a+b}{13} = \dfrac{b+c}{11} = \dfrac{c+a}{12}$.

5. What is $\sin A : \sin B : \sin C$ equal to?

(a) $5 : 6 : 7$

(b) $7 : 6 : 5$

(c) $13 : 11 : 12$

(d) $12 : 11 : 13$

6. What is $\cos A : \cos B : \cos C$ equal to?

(a) $7 : 19 : 25$

(b) $11 : 12 : 13$

(c) $7 : 11 : 13$

(d) $12 : 11 : 13$

For the next **two (02)** items that follow :

Given that $\tan\left(\frac{A}{2}+\frac{B}{2}\right)=p$ and $\tan\left(\frac{A}{2}-\frac{B}{2}\right)=q$, where $pq \neq \pm 1$.

7. What is $\tan A$ equal to?

(a) $\frac{p-q}{1+pq}$

(b) $\frac{p+q}{1-pq}$

(c) $\frac{p+q}{1+pq}$

(d) $\frac{p-q}{1-pq}$

8. What is $\tan B$ equal to?

(a) $\frac{p-q}{1+pq}$

(b) $\frac{p+q}{1-pq}$

(c) $\frac{p+q}{1+pq}$

(d) $\frac{p-q}{1-pq}$

For the next **two (02)** items that follow :

Let $4(A+B)=\pi$.

9. What is $(1+\tan A)(1+\tan B)$ equal to?

(a) 1

(b) 2

(c) 4

(d) 6

10. What is $(\cot A-1)(\cot B-1)$ equal to?

(a) 1/2

(b) 1

(c) 3/2

(d) 2

11. What is $6\sin(\pi/18)-8\sin^3(\pi/18)$ equal to?

(a) 1/4

(b) 1/2

(c) 1

(d) 2

12. How many values of θ satisfy the equation $\tan^2\theta+\cot^2\theta=2$, where $0^\circ<\theta<360^\circ$?

(a) 1

(b) 2

(c) 4

(d) 8

13. What is $\sin(\tan^{-1}0{\cdot}75)$ equal to?

(a) 1

(b) 4/5

(c) 3/5

(d) 1/2

14. What is $\tan^2\left(\frac{1}{2}\cos^{-1}\frac{1}{3}\right)$ equal to?

(a) 1/2

(b) 1

(c) 2

(d) 4

15. What is

$$\tan(1125°)\cot(405°) + \tan(765°)\cot(675°)$$

equal to?

(a) 2

(b) 1

(c) 0

(d) −2

16. If A is a square matrix of order 3 and $|A| = 2$, then what is adj(adj$A$) equal to?

(a) $8A$

(b) $4A$

(c) $2A$

(d) $2I$, where I is the identity matrix of order 3

17. If A is a square matrix of order 3 such that

$$A(\text{adj}A) = \begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix}$$

then what is $|A|$ equal to?

(a) 4

(b) 8

(c) 16

(d) 64

18. Let

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

What is the least value of θ for which $A + A^T = I$, where I is the identity matrix of order 2?

(a) $\pi/2$

(b) $\pi/3$

(c) $\pi/4$

(d) $\pi/6$

19. Let A, B and C be the square matrices of order 2 such that $AB = AC$. Which of the statements given below is/are correct?

I. B and C are not necessarily equal if A is a singular matrix.

II. B and C are equal if A is a non-singular matrix.

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

20. If A is a square matrix of order 3 and $|A| = 4$, then what is $|\text{adj}A|$ equal to?

(a) 64

(b) 16

(c) 8

(d) 4

21. Let A and B be symmetric matrices of the same order. Which of the following statements is/are correct?

I. $(AB - BA)$ is also a symmetric matrix.

II. $(BA - AB)$ is a skew-symmetric matrix.

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

22. If $A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$, then what is $|A^{-1}|$ equal to?

(a) -2

(b) -1

(c) $-1/2$

(d) 2

23. Let S be the set of all real numbers and R be a relation on S defined by $xRy \Rightarrow |x| \leq y$. Then R is

(a) reflexive only

(b) transitive only

(c) both reflexive and transitive

(d) neither reflexive nor transitive

24. 4-digit numbers are formed with 1, 2, 3 and 4. What is the number of the 4-digit numbers in which at least one digit is repeated?

(a) 256

(b) 244

(c) 232

(d) 220

25. Given

$S = \{(x, y, z) \in R^3 : xyz = 210,\ x < y < z,$ where x, y, z are positive integers$\}$

What is the total number of elements in S?

(a) 10

(b) 12

(c) 13

(d) More than 13

26. If $x = 4 + i$, where $i = \sqrt{-1}$, then what is $x^3 + 2x^2 - 63x + 171$ equal to?

(a) -1

(b) 0

(c) 1

(d) 2

27. Let α and β be the roots of the equation

$$x^2 - p(x+1) - q = 0$$

What is

$$\frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+q} + \frac{\beta^2+2\beta+1}{\beta^2+2\beta+q}$$

equal to?

(a) -1

(b) 0

(c) 1

(d) 2

28. If the sum of the roots of the equation

$$\frac{1}{2x+p} + \frac{1}{2x+q} = \frac{1}{r}$$

is zero, then what is r equal to?

(a) $p-q$

(b) $p+q$

(c) $(p-q)/2$

(d) $(p+q)/2$

29. Let α and β be the roots of the equation $ax^2+bx+c=0$ and $p_n = \alpha^n + \beta^n$, where $n > 1$. What is

$$ap_{n+1} + bp_n + cp_{n-1}$$

equal to?

(a) 0

(b) 1

(c) $a+b+c$

(d) $a-b+c$

30. If p, q and r are in AP, then what is

$$p^3 + 3pr(p+r) + r^3$$

equal to?

(a) q^3

(b) $2q^3$

(c) $3q^3$

(d) $8q^3$

31. If $\frac{1}{b-a}+\frac{1}{b-c}=\frac{2}{b}$, then a, b and c are in

(a) AP

(b) GP

(c) HP

(d) Neither in AP nor in GP nor in HP

32. Let P, Q, R and S be the sum of n terms, $2n$ terms, $3n$ terms and $4n$ terms respectively of an AP such that $Q = 3P$. Which one of the following is correct?

(a) $5R = 3S$

(b) $6R = 5S$

(c) $8R = 7S$

(d) $9R = 8S$

33. In a GP, the 3rd, 5th and 7th terms are x, x^2+2 and x^3+10 respectively, where $x > 1$. What is the 8th term?

(a) 18

(b) $18\sqrt{3}$

(c) 54

(d) $54\sqrt{3}$

34. If $(1+i)^n - 16 = 0$, where $i = \sqrt{-1}$ and n is a positive integer, then what is the least value of n?

(a) 2

(b) 4

(c) 8

(d) 16

35. What is $i \times i^4 \times i^9 \times i^{16} \times \ldots \times i^{576}$, where $i = \sqrt{-1}$, equal to?

(a) $-i$

(b) -1

(c) 1

(d) i

36. If $z = x + iy$ be such that $\left|\frac{z+\lambda i}{z-\lambda i}\right| = 1$, where $i = \sqrt{-1}$ and λ is a positive real number, then which of the following statements is/are correct?

I. z lies on the line $y = x$.

II. The amplitude of z is $\frac{\pi}{4}$.

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

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37. What is

$$\begin{vmatrix} \frac{1}{a} & bc & a^3 \\ \frac{1}{a}+\frac{1}{b} & c(a+b) & a^3+b^3 \\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c} & ab+bc+ca & a^3+b^3+c^3 \end{vmatrix}$$

equal to?

(a) 0

(b) 3

(c) $(a+b+c)(ab+bc+ca)$

(d) $a^3+b^3+c^3-3abc$

38. If

$ae+bg=cf+dh=p,\ af+bh=ce+dg=q$

then what is

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \times \begin{vmatrix} e & f \\ g & h \end{vmatrix}$$

equal to?

(a) p^2q^2

(b) pq

(c) p^2+q^2

(d) p^2-q^2

39. If

$x+ay+a^2z=1,\ x+by+b^2z=1,\ x+cy+c^2z=1$

where $a \neq b \neq c$, then what is $x+y+z$ equal to?

(a) 0

(b) 1

(c) 3

(d) $a+b+c$

40. Let

$$A=\begin{pmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{pmatrix} \text{ and } B=\begin{pmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{pmatrix}$$

If $C=AB^{-1}$, then what is $\det(C)$ equal to?

(a) $\sec^2\theta$

(b) $\sec^4\theta$

(c) $\cos^2\theta$

(d) 1

41. What is the LCM of $(11110)_2$, $(10100)_2$ and $(11011)_2$?

(a) $(1000001100)_2$

(b) $(1000011100)_2$

(c) $(1000111100)_2$

(d) $(1000011101)_2$

42. What is the coefficient of x^{2n} in the expansion of the following?

$$(1+x)^{2n}\left(1+\frac{1}{x}\right)^{2n}$$

(a) 1

(b) $2n(2n-1)(2n-2)\ldots(n+1)n$

(c) $4n(4n-1)(4n-2)\ldots(2n+1)(2n)$

(d) Such term does not exist

43. Consider the following statements in respect of the expansion of $(x^2+x^{-2}+2)^4$:

I. The number of terms in the expansion is 5.

II. One of the coefficients in the expansion has a maximum value equal to 70.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

44. Consider the following relations from A to B, where $A = \{1, 3, 5\}$ and $B = \{2, 4, 6, 8\}$:

I. $\{(1, 2), (3, 2), (3, 6), (5, 8)\}$

II. $\{(3, 4), (5, 8), (1, 6), (3, 2)\}$

III. $\{(1, 2), (3, 6)\}$

IV. $\{(1, 6), (3, 2), (5, 2)\}$

Which of the above is/are function(s) from A to B?

(a) I, III and IV

(b) IV only

(c) II and IV

(d) III only

45. If $x^2 - 2x\cos\theta + 1 = 0$, then what is the magnitude of x?

(a) -1

(b) 0

(c) 1

(d) $\sqrt{2}$

46. Consider the following statements in respect of sets A and B :

I. If $x \in A$ and $A \in B$, then $x \in B$.

II. If $A \subset B$ and $x \notin B$, then $x \notin A$.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

47. A research group conducted a survey of 500 consumers and reported that 370 consumers preferred product X and 240 consumers preferred product Y. What is the least number that must have preferred both the products?

(a) 90

(b) 100

(c) 110

(d) Cannot be determined due to insufficient data

48. The Cartesian product $A \times A$ has 25 elements among which are found (3, 1), (6, 2), (5, 3). Which of the following statements is/are correct?

I. It is possible to determine other elements of $A \times A$.

II. $(5, 5) \in A \times A$ and $(1, 3) \notin A \times A$

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

49. Consider the following statements in respect of a relation R from a set A to a set B :

I. The set B is called codomain of the relation R.

II. The range of the relation is always equal to codomain of the relation R.

III. The domain of the relation R must be equal to the set A.

Which of the statements given above is/are correct?

(a) I only

(b) III only

(c) I and II

(d) I and III

50. Consider the following system of linear inequalities :

$$x - y \leq 3 \text{ and } x + y \geq 5$$

The solution of the inequalities lies in

(a) first quadrant only

(b) second quadrant only

(c) both first quadrant and second quadrant only

(d) all the four quadrants

51. A circle touches each of the lines $x - y = 0$ and $x + y = 0$ at unit distance from the origin. The centre of the circle may be at

(a) $(\sqrt{2}, 0)$

(b) $(\sqrt{3}, 0)$

(c) $(1, 0)$

(d) $(1, 1)$

52. A line $(\sin\theta)x + (\cos\theta)y = \sin 2\theta$ cuts the coordinate axes at points P and Q. Let M be the midpoint of the line segment PQ. What is the distance of M from the origin?

(a) 2

(b) 1

(c) $\cos\theta + \sin\theta$

(d) $\cos\theta - \sin\theta$

53. The centre and one of the foci (F) of an ellipse are at $(0, 0)$ and $(-c, 0)$ respectively. If $P(x, y)$ is any point on the ellipse and $2a$ is the length of major axis, then what is PF equal to?

(a) $a - (cx/a)$

(b) $a - (ax/c)$

(c) $a + (ax/c)$

(d) $a + (cx/a)$

54. What is the equation of the line which is equidistant from the lines $2x - 4y - 7 = 0$ and $6x - 12y + 1 = 0$?

(a) $12x - 24y - 21 = 0$

(b) $12x - 24y - 19 = 0$

(c) $3x - 6y - 5 = 0$

(d) $4x - 8y - 3 = 0$

55. A ray of light passing through the point $P(1, 2)$ reflects on the x-axis at point N and the reflected ray passes through the point $Q(5, 3)$. What is the distance of the point N from the origin?

(a) 3 units

(b) 14 / 5 units

(c) 13 / 5 units

(d) 2 units

56. A line cuts off intercept p on the x-axis and intercept q on the y-axis, where $p > q$. The sum of the intercepts is 2 and the product of the intercepts is -15. What is the equation of the line?

(a) $3x - 5y - 15 = 0$

(b) $3x - 5y + 15 = 0$

(c) $5x - 3y - 15 = 0$

(d) $5x + 3y - 15 = 0$

57. Let P and Q be the points on the positive x-axis and positive y-axis respectively. A point $N(2, 1)$ divides the line segment PQ in the ratio 1 : 2. What is the equation of the line?

(a) $2x + y - 5 = 0$

(b) $x + y - 3 = 0$

(c) $x + 2y - 4 = 0$

(d) $x - y - 1 = 0$

58. $ABCD$ is a square. The equations of AB, AD and BD are $y = 0$, $x = 0$ and $x + y - 4 = 0$ respectively. What is the equation of AC?

(a) $x + y = 0$

(b) $x - y = 0$

(c) $x - y - 4 = 0$

(d) $x - y + 4 = 0$

59. ABC is a triangle, where the vertices A and B are fixed points both lying on the x-axis. Let $AB = 10$ cm. The vertex C moves such that $\frac{1}{\tan A} + \frac{1}{\tan B} = \frac{1}{k}$, where $k \neq 0$. What is the equation of the locus of the point C?

(a) $y = 10k$

(b) $x = 10k$

(c) $ky = 10$

(d) $kx = 10$

60. What is the eccentricity (e) of the parabola $4x^2 + y = 0$?

(a) $e < 0$

(b) $0 < e < 1$

(c) $e = 1$

(d) $e > 1$

61. The vertices of a triangle are $A(2, 0, 0)$, $B(0, 6, 0)$ and $C(0, 0, 4)$. If AD, BE and CF are the medians of the triangle, then what is $AD^2 + BE^2 + CF^2$ equal to?

(a) 81

(b) 84

(c) 87

(d) 90

62. A line makes angles α, β and γ with the positive directions of x-axis, y-axis and z-axis respectively such that $\alpha + \beta = 90°$. Which of the following statements is/are correct?

I. The maximum value of $\cos\alpha + \cos\beta$ is $\sqrt{2}$.

II. The minimum value of $\cos\alpha + \cos\beta + \cos\gamma$ is 1.

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

63. What is the area of the triangle whose vertices are (0, 7, 10), (−1, 6, 6) and (−4, 9, 6)?

(a) 9 square units

(b) 12 square units

(c) 15 square units

(d) 18 square units

64. If O is the origin and P is the point (2, −4, 6), then what is the equation of the plane through P and perpendicular to OP?

(a) $x + 2y + 3z - 28 = 0$

(b) $x - 2y + 3z - 28 = 0$

(c) $x - 2y + 3z + 28 = 0$

(d) $2x - 4y + 6z - 55 = 0$

65. The image of the point $P(1, 3, 4)$ with respect to the plane $Ax + By + Cz + D = 0$ is (−3, 5, 2). If $A + B + C = 2$, then what is D equal to?

(a) −3

(b) 3

(c) 6

(d) D cannot be determined due to insufficient data

66. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ be the vectors. Consider the following :

I. $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$

II. $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})$

III. $(\vec{a}\cdot\vec{b})\cdot(\vec{c}\cdot\vec{d})$

IV. $(\vec{a}\cdot\vec{b})\times(\vec{c}\cdot\vec{d})$

V. $\{(\vec{a}\times\vec{b})\cdot\vec{c}\}\times\vec{d}$

where '$\cdot$' represents dot product and '$\times$' represents cross product of vectors. How many of the above are ***not*** well-defined?

(a) One

(b) Two

(c) Three

(d) Four

67. The scalar projection of $\vec{a}=\lambda\hat{i}+\hat{j}-2\hat{k}$ on $\vec{b}=2\hat{i}-\hat{j}-\lambda\hat{k}$ is $7/3$. What is the value of λ?

(a) 1

(b) 2

(c) 3

(d) 4

68. How much angle does

$$\vec{r}=(\cos\theta)\hat{i}+(\sin\theta)\hat{j}+\hat{k}$$

make with the positive direction of z-axis?

(a) $\pi/2$

(b) $\pi/3$

(c) $\pi/4$

(d) $\pi/6$

69. Let $\vec{a}$ and $\vec{b}$ be unit vectors inclined at $30°$. What is the area of the parallelogram whose sides are represented by the vectors $\vec{a}+3\vec{b}$ and $3\vec{a}+\vec{b}$?

(a) $8\sqrt{3}$ square units

(b) 8 square units

(c) $4\sqrt{3}$ square units

(d) 4 square units

70. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=0$. If the angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$, then what is $\vec{a}$ equal to?

(a) $(\vec{b}\times\vec{c})$

(b) $\sqrt{2}(\vec{b}\times\vec{c})$

(c) $2(\vec{b}\times\vec{c})$

(d) $\sqrt{3}(\vec{b}\times\vec{c})$

71. What is $\int \frac{\sqrt{x}}{\sqrt{1-x^3}} dx$ equal to?

(a) $\frac{x^{3/2}}{3} + c$

(b) $\frac{2x^{3/2}}{3} + c$

(c) $\frac{1}{3}\sin^{-1}(x^{3/2}) + c$

(d) $\frac{2}{3}\sin^{-1}(x^{3/2}) + c$

72. What is $\int_0^1 \frac{dx}{(ax+bx+c)^2}$ equal to?

(a) $-\frac{1}{a+b+c}$

(b) $-\frac{1}{c(a+b+c)}$

(c) $\frac{1}{a+b+c}$

(d) $\frac{1}{c(a+b+c)}$

73. What is $\int e^{x \ln 10} e^x dx$ equal to?

(a) $\frac{(10e)^x}{1+\ln 10} + c$

(b) $\frac{10e^x}{1+\ln 10} + c$

(c) $\frac{e^{2x}}{2} + c$

(d) $\frac{e^{2x}}{4} + c$

74. What is $\int_1^2 x^x(1+\ln x)\,dx$ equal to?

(a) 4

(b) 3

(c) 2

(d) 1

75. If $f(x+y) = f(x) + f(y)$, then what is

$$\int_{-1}^{1} f(x)dx$$

equal to?

(a) 0

(b) $\int_0^1 f(x)dx$

(c) $2\int_0^1 f(x)dx$

(d) $2f(1)$

76. What is the area bounded by $y = e^{|x|}$ and the lines $|x| = 1$ and $y = 0$?

(a) $(e-1)$ square units

(b) $(2e-2)$ square units

(c) $2e$ square units

(d) $(2e-1)$ square units

77. The slope of the tangent to the curve $y = f(x)$ at $(x, f(x))$ is $2x$. If the curve passes through the origin, then what is the area bounded by the curve, the x-axis and the line $x = 1$?

(a) 1/3 square unit

(b) 2/3 square unit

(c) 1 square unit

(d) 4/3 square units

78. What is the solution of

$y\,dx - x\,dy - y^2 \cos x\,dx = 0$?

(a) $y = \dfrac{x}{c + \sin x}$

(b) $y = \dfrac{x}{c - \sin x}$

(c) $y = \dfrac{x}{1 + c\sin x}$

(d) $y = \dfrac{x}{1 - c\sin x}$

79. What is the differential equation of the family of straight lines passing through the origin?

(a) $y\dfrac{dy}{dx} - x = 0$

(b) $y\dfrac{dy}{dx} + x = 0$

(c) $x\dfrac{dy}{dx} - y = 0$

(d) $x\dfrac{dy}{dx} + y = 0$

80. What is the degree of the differential equation representing the family of curves $y^2 = \sqrt{c}\,x$, where c is a positive parameter?

(a) 1

(b) 2

(c) 3

(d) 4

81. What is the domain of the function $f(x) = \log_x e$?

(a) $(0, \infty)$

(b) $(0, \infty) - \{1\}$

(c) $(0, \infty) - \{e\}$

(d) $(0, \infty) - \{10\}$

82. If $f(x)$ is the integral of

$$\frac{1 + \cos^2 x - 2\cos x}{x \sin^2 x \tan x}$$

then what is $\lim\limits_{x \to 0} f'(x)$ equal to?

(a) 0

(b) 1/2

(c) 1/3

(d) 1/4

83. Consider the following statements in respect of the function $f(x) = [3x]$, where $[\cdot]$ is the greatest integer function :

I. $f(x)$ is continuous at $x = 1/3$.

II. $f(x)$ is differentiable at $x = 1/4$.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

84. What is $\lim\limits_{x \to 0} \dfrac{\sin^2 x}{x|x|}$ equal to?

(a) -1

(b) 0

(c) 1

(d) Limit does not exist

85. Let $f(x) = p|x| + q$, where p and q are real constants. If $f'(x)$ exists at $x = 0$, then what is the value of p?

(a) -1

(b) 0

(c) 1

(d) q

86. Consider the following statements in respect of the given function :

$$f(x) = \begin{cases} \dfrac{x^3}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

I. $f(x)$ is continuous everywhere.

II. $f(x)$ is differentiable everywhere.

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

87. What is $\lim_{x \to \infty} (\sqrt{x + \sqrt{x}} - \sqrt{x})$ equal to?

(a) $-1/2$

(b) 0

(c) $1/2$

(d) 1

88. If m and n are the roots of the equation $x^2 - px + q = 0$, then what is

$$\lim_{x \to m} \frac{e^{x^2 - px + q} - 1}{(x - m)(x - n)}$$

equal to?

(a) -1

(b) 0

(c) 1

(d) e

89. Let $f(x) = x^n + k$, where n is a natural number and k is a positive real constant such that $f(x) + f\left(\frac{1}{x}\right) = f(x) f\left(\frac{1}{x}\right)$ and $f(2) = 9$. What is $f(-1)$ equal to?

(a) -2

(b) -1

(c) 0

(d) 1

90. If $f(x) = \cos\left\{\frac{\pi}{3}[x] + x\right\}$ for $1 < x < 2$, where $[\cdot]$ is the greatest integer function, then what is $f\left(\frac{\pi}{2}\right)$ equal to?

(a) -1

(b) $-\sqrt{3}/2$

(c) $-1/2$

(d) $1/2$

91. Consider the following statements :

I. $\frac{d}{dx}\ln|x| = -\frac{1}{x}$ if $x < 0$

II. $\frac{d}{dx}\ln|x| = \frac{1}{x}$ if $x > 0$

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

92. If $y = \left|\sin\left(\frac{\pi}{4} - x\right)\right|$, then what is $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ equal to?

(a) -1

(b) 0

(c) 1

(d) It does not exist

93. Let f be a differentiable function such that $f(x+y) = f(x) + f(y)$ for all $x, y \in R$. Which of the following statements is/are correct?

I. If $f(x) = x\,g(x)$, then the derivative of $f(x)$ with respect to x is equal to $g(0)$.

II. If $f(x) = x^2 g(x)$, then the derivative of $f(x)$ with respect to x is equal to 0.

(Here g is a continuous function)

Select the answer using the code given below.

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

94. If

$$f(x) = x^{n-1} + x^{n-2} + x^{n-3} + \ldots + 1$$

then what is $f'(2)$ equal to?

(a) $n2^{n-1} - 2^n + 1$

(b) $n2^n - 2^{n-1} + 1$

(c) $n2^{n-1} - 2^n - 1$

(d) $(n-1)2^{n-2} - 2^{n-1} + 1$

95. ABC is a triangle right angled at B. If $AC = 8$ units, then what is the area of the triangle of maximum area?

(a) 32 square units

(b) 24 square units

(c) 16 square units

(d) 12 square units

96. For the curve $y = xe^{2x}$

(a) minimum occurs at $x = -2$

(b) minimum occurs at $x = -1$

(c) minimum occurs at $x = -1/2$

(d) maximum occurs at $x = -1/2$

97. Let $f(x) = \int xe^x dx$. Then $f(x)$ decreases in the interval

(a) $(-\infty, 0)$

(b) $(0, \infty)$

(c) $(-\infty, -1)$

(d) $(-1, \infty)$

98. What is the solution of the differential equation $\frac{dy}{dx} = 1 + x\cot(y - x)$?

(a) $\sec(y - x) = ce^{\frac{x^2}{2}}$

(b) $\cos(y - x) = ce^{\frac{x^2}{2}}$

(c) $\text{cosec}(y - x) = ce^{\frac{x^2}{2}}$

(d) $\sin(y - x) = ce^{\frac{x^2}{2}}$

99. Consider the following statements in respect of the function

$$f : R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{3}{5}\right\}$$

such that $f(x) = \frac{3x + 2}{5x - 3}$:

I. $f(x)$ is a bijective function.

II. $f^{-1}(x) = f(x)$

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

100. Let A and B be the sets having only 2 and 3 elements respectively. What is the total number of mappings from A to B?

(a) 4

(b) 6

(c) 8

(d) 9

101. The variance of 10 observations is 25. If 2 is multiplied to each of the observations and subsequently 7 is subtracted from each multiplied observation, then what is the new variance?

(a) 125

(b) 100

(c) 50

(d) 25

102. If the random variable X has mean 5 and standard deviation 4, then what is the standard deviation of the random variable $Y = 3X + 4$?

(a) 16

(b) 12

(c) 10

(d) 6

103. For a binomial distribution with mean 4 and standard deviation $\sqrt{3}$, what is the value of $P(X = 0)$?

(a) $\left(\frac{1}{4}\right)^{16}$

(b) $\left(\frac{3}{4}\right)^{16}$

(c) $\left(\frac{1}{4}\right)^{12}$

(d) $\left(\frac{3}{4}\right)^{12}$

104. If the correlation coefficient between the variables X and Y is zero, then the two lines of regression are

(a) $X + Y + 11 = 0$ and $2X + 3Y + 4 = 0$

(b) $X = \bar{X}$ and $Y = \bar{Y}$

(c) $X = 0 \cdot 4Y + 10$ and $Y = 1 \cdot 6X + 20$

(d) $\bar{X} + \bar{Y} + 11 = 0$ and $2\bar{X} + 3\bar{Y} + 4 = 0$

105. Four dice are rolled. What is the probability of getting a total of the numbers on the dice as 7?

(a) 5/1296

(b) 7/324

(c) 7/1296

(d) 5/324

106. If $P(A \cap B) = 1/2$ and $P(\bar{A} \cap \bar{B}) = 1/2$, and $2P(A) = P(B) = k$, then what is the value of k?

(a) 1/4

(b) 1/2

(c) 1/3

(d) 2/3

107. A committee has 6 men and 4 women. One member is selected randomly which is woman only. What is the probability that she belongs to a subgroup of 2 senior women?

(a) 1/3

(b) 1/2

(c) 2/3

(d) 3/4

108. The standard deviation of X is 4 and the standard deviation of Y is 5. If the correlation coefficient r is 0·8, then what is the regression coefficient of Y on X?

(a) −2

(b) −1

(c) 0

(d) 1

109. In a class, the probability of passing students in Mathematics is 0·7 and the probability of passing both Mathematics and Statistics is 0·5. What is the probability that a student passes Statistics given that the student passed Mathematics?

(a) 5/7

(b) 4/7

(c) 3/7

(d) 2/7

110. If the mean of 50 observations is 40 and the standard deviation is 8, then what is the coefficient of variation?

(a) 15%

(b) 20%

(c) 25%

(d) 30%

111. If two fair dice are tossed, then what is the probability that the sum of the numbers on the faces of the dice is neither 5 nor 7?

(a) 11/18

(b) 2/3

(c) 13/18

(d) 5/6

112. Consider the following statements for three events A, B and C :

I. $P(A \cap B \cap C) \leq P(A) + P(B) + P(C) - 2$

II. $P(A \cup B \cup C) \geq P(A) + P(B) + P(C)$

Which of the statements given above is/are correct?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

113. A Mathematics problem is given to two students X and Y to solve. The odds in favour of X solving the problem are 6 to 9 and the odds against Y in solving the problem are 6 to 5. What is the probability that the problem will be solved if both X and Y try to solve the problem?

(a) 27/55

(b) 31/55

(c) 37/55

(d) 39/55

114. The events A and B are independent among three events A, B and D. If $P(A \cap B \cap D) = 0.04$, $P(D|A \cap B) = 0.25$ and $P(B) = 4P(A)$, then what is the value of $P(A \cup B)$?

(a) 0.88

(b) 0.84

(c) 0.28

(d) 0.16

For the next **three (03)** items that follow :

Let A, B, C and D be mutually exclusive and exhaustive events such that

$$\frac{P(A)}{6} = \frac{P(B)}{3} = \frac{P(C)}{4} = \frac{P(D)}{2}$$

115. What is $\frac{P(A) + 2P(B)}{3P(C) + P(D)}$ equal to?

(a) 5/4

(b) 6/7

(c) 2/5

(d) 17/6

116. If G is the geometric mean of $P(A)$, $P(B)$, $P(C)$ and $P(D)$, then what is G equal to?

(a) $\frac{5}{2\sqrt{3}}$

(b) $\frac{2}{5\sqrt{3}}$

(c) $\frac{\sqrt{3}}{5}$

(d) $\frac{1}{5\sqrt{3}}$

117. If H is the harmonic mean of $P(A)$, $P(B)$, $P(C)$ and $P(D)$, then what is H equal to?

(a) 14 / 75

(b) 15 / 76

(c) 16 / 75

(d) 16 / 15

For the next **three (03)** items that follow :

Five numbers are randomly picked from the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and arranged them in ascending order $[x_1 < x_2 < x_3 < x_4 < x_5$; x_1 being the smallest number and x_5 being the largest number selected].

118. What is $P(x_1 = 4)$ equal to?

(a) 5 / 84

(b) 1 / 14

(c) 1 / 7

(d) 19 / 84

119. What is $P(x_3 = 6)$ equal to?

(a) 5 / 21

(b) 10 / 21

(c) 11 / 21

(d) 17 / 21

120. What is $P(x_1 = 2, x_2 = 3, x_3 = 8)$ equal to?

(a) 1 / 12

(b) 1 / 60

(c) 1 / 252

(d) 0