

(5)

1. Let p, q and r be three unequal numbers such that p, q and r are in AP. If $(q - p), (r - q)$ and p are in GP, then $(p + q) : (q + r) : (r + p)$ equals

- (a) $1 : 2 : 3$ $p = a - d$
 $q = a$
 $r = a + d$
- (b) $3 : 4 : 5$
- (c) $3 : 5 : 4$
- (d) $1 : 3 : 2$

2. If p, g_1, g_2 and q are in GP and m is the arithmetic mean of p and q , then

$\frac{g_1^2}{g_2} + \frac{g_2^2}{g_1}$ is equal to

- (a) m
- (b) $2m$
- (c) 1
- (d) $\frac{1}{2}$

3. Consider the following inequalities :

- I. $1 + 4i > 3 + 2i$
- II. $2 + 3i < 3 + 4i$ ✓
- III. $4 + 3i > 3 + 4i$

where $i = \sqrt{-1}$

How many of the above are valid ?

- (a) None
- (b) One
- (c) Two
- (d) All the three

4. Let Z_1 and Z_2 be complex numbers such that $\frac{3Z_1}{4Z_2}$ is purely imaginary.

What is $\left| \frac{Z_1 + Z_2}{Z_1 - Z_2} \right|$ equal to ?

- (a) 2
- (b) $\frac{3}{2}$
- (c) $\frac{5}{4}$
- (d) 1

5. If α, β, γ are cube roots of -8 , then

what is $\frac{\alpha^2 p^2 + \beta^2 q^2 + \gamma^2 r^2}{\beta^2 p^2 + \gamma^2 q^2 + \alpha^2 r^2}$ equal to ?

- (a) $\frac{\gamma}{\alpha}$
- (b) $\frac{\gamma}{\beta}$
- (c) $\frac{2\gamma}{\alpha}$
- (d) $\frac{2\gamma}{\beta}$

6. The sum of the first n terms of an AP is $3n^2 + 5n$. If the m^{th} term of the AP is 68, then what is the value of m ?

- (a) 9
- (b) 10
- (c) 11
- (d) 12

$S_n = \frac{n}{2}(2a + (n-1)d)$

7. A set S contains $(2n + 1)$ elements. If the number of subsets of S which contain at most n elements is 1024, then what is the value of n ?

- (a) 10
- (b) 8
- (c) 6
- (d) 5

Handwritten notes for Q7: $2^n + 2^n$, 2^{n+1} , $1024 = 2^{10}$, $2^{n+1} = 2^{10}$, $n+1 = 10$, $n = 9$. A circle with '1' inside and '2' below it is also present.

11. If the highest degree coefficient is equal to 1, then what is the total number of quadratic equations which are unchanged on squaring their roots?

- (a) 6
- (b) 4
- (c) 2
- (d) None

8. What is the maximum number of points of intersection of 5 circles?

- (a) 10
- (b) 15
- (c) 20
- (d) 25

Handwritten notes for Q8: $2 \cdot nC_2$, $2 \cdot 5C_2$, $\frac{5 \times 4}{2}$. A diagram shows two overlapping circles.

12. Let α and β be the roots of the quadratic equation $x^2 - 2bx + c^2 = 0$ where b, c are positive real numbers. Let A be the arithmetic mean of α and β ; and G be the geometric mean of α and β . What are the roots of the quadratic equation $x^2 - (b + c)x + bc = 0$?

- (a) A, G
- (b) $2A, G$
- (c) $A, 2G$
- (d) $2A, 2G$

9. What is the greatest value of r satisfying the inequality ${}^{15}C_{r+1} > 2 \times {}^{15}C_r$?

- (a) 2
- (b) 3
- (c) 4
- (d) 5

Handwritten notes for Q9: $\frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} > 2 \times \frac{15 \times 14 \times 13 \times 12}{3 \times 2 \times 1}$

10. If $n = {}^m C_2$ then what is ${}^n C_2$ equal to?

- (a) ${}^{m+1} C_4$
- (b) $2 \times {}^{m+1} C_4$
- (c) $3 \times {}^{m+1} C_4$
- (d) ${}^{m+2} C_4$

Handwritten notes for Q10: ${}^m C_2 = {}^n C_2$, ${}^m C_2 = {}^n C_2$

13. If $1 - \log_{10} 2 = \log_{10}(5^x + 4^x + 3^x + 2^x + 1)$, then what is a value of x ?

- (a) 10
- (b) 5
- (c) 1
- (d) 0

14. Let $f(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ q & q^2 & q^3 \end{vmatrix}$

where q is any constant, then what is

$\frac{d^2}{dx^2}(f(x))$ at $x = 0$ equal to ?

- (a) -1
- (b) 0
- (c) 1
- (d) q

15. If $\begin{vmatrix} a-b & p-q & x-y \\ b-c & q-r & y-z \\ c-a & r-p & z-x \end{vmatrix} = k \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$,

then what is the value of k ?

- (a) -1
- (b) 0
- (c) $\frac{1}{2}$
- (d) 1

16. If p, q, r are the cube roots of unity, then what is

$\begin{vmatrix} p^2+q^2 & r^2 & r^2 \\ p^2 & q^2+r^2 & p^2 \\ q^2 & q^2 & r^2+p^2 \end{vmatrix}$ equal to ?

- (a) -1
- (b) 0
- (c) 1
- (d) 4

$p = 1$
 $q = \omega$
 $r = \omega^2$
 $1 + \omega^2 = \omega^4$
 $\omega^2 + \omega^4 = 1$

17. If A is a square matrix such that $|A| = -2$, then $|AA^T|$, where A^T is the transpose of A , is equal to

- (a) -4
- (b) 1
- (c) 2
- (d) 4

~~$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$~~ $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$
 $\frac{1}{2 \times 2} \times$

18. Consider the following statements :

- I. If $n \times n$ ($n > 1$) matrix is symmetric, then its inverse is also a symmetric matrix.
- II. If $n \times n$ ($n > 1$) matrix is singular, then its adjoint is also a singular matrix.

Which of the statements given above is/are correct ?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

19. If $M = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then what is the

value of $|M| |\text{adj}M|$?

- (a) 8
- (b) 64
- (c) 256
- (d) 512

$(M) = 2(4-0) = 8$
 $8 \times 8 = 64$

20. If $M_k = \begin{bmatrix} k & k-1 \\ k-1 & k \end{bmatrix}$ where k is a natural number, then what is $|M_1| + |M_2| + |M_3| + \dots + |M_{50}|$ equal to?

- (a) 50
- (b) 1250
- (c) 2500
- (d) 5000

Handwritten note: $\begin{bmatrix} 50 & 51 \\ 50 & 50 \end{bmatrix}$

21. If M^T is the transpose of a 2×2 matrix M , then which of the following is/are correct?

- I. $|M + M^T| = |M| + |M^T|$ if M is symmetric.
- II. $|M + M^T| = 0$ if M is anti-symmetric.

Select the answer using the code given below:

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

Handwritten note: $\begin{pmatrix} 4 & 2 \\ 4 & 2 \end{pmatrix}$

22. If M is a square matrix such that $M^3 = M$, then how many values of $|M|$ are possible?

- (a) One
- (b) Two
- (c) Three
- (d) Four

Handwritten note: $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$
 $|M| =$

23. Let $p = (x + y + z)$ and $q = xyz$. If

$\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix}$ is positive, then which one

of the following is correct?

- (a) $q > p$
- (b) $q + 1 > p$
- (c) $q + 2 > p$
- (d) $q + 2 \geq p$

Handwritten note: $a = xyz$

24. What is $((A \cap B) \cup (A - B)) - ((A \cap B) \cup (B - A)) \cup A$ equal to?

- (a) ϕ
- (b) A
- (c) B
- (d) $A \cup B$

25. Let A and B be two sets. For some set C , both $A \cap C$ and $B \cap C$ are empty sets and $A \cup C = B \cup C$. Which of the following is/are true?

- I. $C = \phi$
- II. $A = B$
- III. $A \cup B = C$

Select the answer using the code given below:

- (a) I only
- (b) II only
- (c) I and II only
- (d) I, II and III

26. If $2 \sec 4\beta = \tan 2\alpha + \cot 2\alpha$, then which one of the following is a possible value of $(\alpha + \beta)$?

- (a) $\frac{\pi}{2}$
- (b) $\frac{\pi}{4}$
- (c) $\frac{\pi}{6}$
- (d) $\frac{\pi}{8}$

27. If α and β are complementary angles

such that $\alpha - \beta = \frac{\pi}{6}$ and $m \tan \beta =$

$n \tan \alpha$, then what is $\left(\frac{m+n}{m-n}\right)$ equal to?

- (a) 2
- (b) $\frac{2}{\sqrt{3}}$
- (c) 1
- (d) $\frac{1}{\sqrt{3}}$

28. If $x = \sec \theta - \tan \theta$ and $y = \operatorname{cosec} \theta + \cot \theta$, then which one of the following is correct?

- (a) $x + y - xy - 1 = 0$
- (b) $x - y + xy + 1 = 0$
- (c) $x + y + xy - 1 = 0$
- (d) $x - y + xy - 1 = 0$

29. If $\cos \theta = \frac{1}{3}$, then what is the value of

$$\sin \left(\frac{\theta}{2}\right) \sin \left(\frac{3\theta}{2}\right) ?$$

- (a) $\frac{5}{9}$
- (b) $\frac{7}{9}$
- (c) $\frac{10}{9}$
- (d) $\frac{11}{9}$

30. $\cos x + \sqrt{3} \sin x$ is maximum when x is equal to

- (a) $\frac{\pi}{2}$
- (b) $\frac{\pi}{3}$
- (c) $\frac{\pi}{4}$
- (d) $\frac{\pi}{6}$

31. If θ lies in the fourth quadrant and $3 \cot \theta + 4 = 0$, then what is the value of $\sin 2\theta + \cos 2\theta$?

- (a) $-\frac{31}{25}$
- (b) $-\frac{17}{25}$
- (c) 0
- (d) 1

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32. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, $\alpha \neq \beta$ then what is a value of $\cos 2\alpha + \cos 2\beta + 2 \cos(\alpha + \beta)$?

- (a) 0
- (b) 1
- (c) 2
- (d) 4

33. In a triangle ABC , $\sin A = \cos B + \cos C$ then what is $\tan\left(\frac{B}{2}\right) + \cot\left(\frac{B}{2}\right)$ equal to ?

- (a) 1
- (b) $\sqrt{2}$
- (c) $\sqrt{3}$
- (d) 2

$\sin A = \cos B + \cos C$
 $\sin A = \frac{b^2 + c^2 - a^2}{2bc} + \frac{c^2 + a^2 - b^2}{2ca}$

34. Consider the following in respect of inverse circular functions :

- I. $\sin^{-1}(-x) = -\sin^{-1} x$ ✓
- II. $\cos^{-1}(-x) = \cos^{-1} x$ ✓
- III. $\tan^{-1}(-x) = \pi - \tan^{-1} x$
- IV. $\cot^{-1}(-x) = \pi - \cot^{-1} x$

How many of the above are correct ?

- (a) One
- (b) Two
- (c) Three
- (d) All the four

35. What is $\tan\left[2\tan^{-1}\frac{1}{2} - \frac{\pi}{4}\right]$ equal to ?

- (a) -7
- (b) 0
- (c) $\frac{1}{5}$
- (d) $\frac{1}{7}$

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36. The angles A, B and C of a triangle are in the ratio $1 : 1 : 4$. If the longest side of the triangle is 3 units, then what is the perimeter of the triangle ?

- (a) $3 + \sqrt{3}$ units
- (b) $3 + 2\sqrt{3}$ units
- (c) $3 + 3\sqrt{3}$ units
- (d) $6 + \sqrt{3}$ units

$3 \times \frac{1}{2}$

37. In a triangle ABC , if a, b and c are the lengths of the sides opposite to the angles A, B and C respectively, then what is $\frac{\sin(A-B)}{\sin(A+B)}$ equal to ?

- (a) $\frac{a^2}{a^2 - b^2}$
- (b) $\frac{a^2}{a^2 + b^2}$
- (c) $\frac{a^2 - b^2}{c^2}$
- (d) $\frac{a^2 - b^2}{b^2}$

$\frac{a^2}{a^2 + b^2}$

38. What is the smallest positive x satisfying $\log_{\sin x} \cos x + \log_{\cos x} \sin x = 2$?

- (a) $\frac{\pi}{2}$
- (b) $\frac{\pi}{3}$
- (c) $\frac{\pi}{4}$
- (d) $\frac{\pi}{6}$

39. A plane is observed to be approaching the airport. It is at a distance of 10 km from the point of observation and makes an angle of elevation of 67.5° . What is the height of the plane above the ground?

- (a) $10\sqrt{2+\sqrt{2}}$ km
- (b) $10\sqrt{2-\sqrt{2}}$ km
- (c) $5\sqrt{2+\sqrt{2}}$ km
- (d) $5\sqrt{2-\sqrt{2}}$ km

40. What is the length of the chord of a unit circle which subtends at the centre of the circle an angle of 45° ?

- (a) $2\sqrt{2+\sqrt{2}}$ units
- (b) $2\sqrt{2-\sqrt{2}}$ units
- (c) $\sqrt{2+\sqrt{2}}$ units
- (d) $\sqrt{2-\sqrt{2}}$ units

41. Let R be a relation on the set N of natural numbers defined by

$$R = \{(x, y) : x, y \in N \text{ and } x = y^3\}.$$

Which of the following statements is/are correct?

- I. R is symmetric relation
- II. R is transitive relation

Select the answer using the code given below:

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

42. For a given k , what is the minimum value of $x^2 + kx + k^2$?

- (a) 0
- (b) $\frac{k^2}{4}$
- (c) $\frac{3k^2}{4}$
- (d) $\frac{k^2}{2}$

43. Consider the following statements:

- I. $\sqrt{x} + x + 1 = 0$ has two irrational roots.
- II. $5\sqrt{x} - x - 4 = 0$ has two rational roots.

Which of the statements given above is/are correct?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

44. How many numbers greater than 1000 can be formed using the digits 0, 1, 2 and 3 (repetition of digits is not allowed)?

- (a) 24
- (b) 18
- (c) 15
- (d) 12

0, 1, 2, 3



Which of the statements given above is/are correct?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

1001
✓
(B)

45. If p^{th} term of an AP is k , then what is the sum of p^{th} term, $(p + q)^{\text{th}}$ term and $(p - q)^{\text{th}}$ term of the AP?

- (a) $2k$
- (b) $3k$
- (c) $4k$
- (d) $5k$

47. What is the multiplicative inverse of $(\sqrt{2} + 1)^{20}$?

- (a) v
- (b) $v^2 - 1$
- (c) v^2
- (d) $v^2 + 1$

$\frac{1}{(\sqrt{2} + 1)^{20}}$

For the next five (05) items that follow:

Let u be a positive integer and f be a real number lying between 0 and 1.

Further, $(\sqrt{2} + 1)^{10} = u + f$ and

$$(\sqrt{2} - 1)^{10} = v$$

1, 2, 3

46. Consider the following statements:

- I. $(u + v + f)$ is an integer.
- II. $(f + v)$ is an integer.

48. What is the value of $(v + f)$?

- (a) 2
- (b) 1
- (c) 0.5
- (d) 0.25

49. What is the value of u ?

- (a) 9725
- (b) 6971
- (c) 6726
- (d) 6725

46-0

49-1

50. What is the value of uv ?

- (a) 1
- (b) 2
- (c) $0 < uv < 1$
- (d) $1 < uv < 2$

For the next *two (02)* items that follow :

The equation of a straight line which passes through the origin and makes an angle α ($0 \leq \alpha \leq 90^\circ$) with the line $L : x + \sqrt{3}y + 3\sqrt{3} = 0$ is $x - \sqrt{3}y = 0$.

51. What is the value of α ?

- (a) 30°
- (b) 45°
- (c) 60°
- (d) 90°

52. What is the angle made by the line L with positive direction of y -axis ?

- (a) 30°
- (b) 45°
- (c) 60°
- (d) 90°

For the next *two (02)* items that follow :

$P(-2, -3, 5)$, $Q(4, -1, 5)$, $R(6, -4, 8)$ and $S(2, -6, 10)$ are four points.

53. Consider the following statements :

I. PQ is parallel to RS .

II. PR is perpendicular to QS .

Which of the statements given above is/are correct ?

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

54. What is $(PQ^2 + 2QS^2 - 2PR^2)$ equal to ?

- (a) -1
- (b) 0
- (c) 1
- (d) 2

55. For the next *two (02)* items that follow :

A line L passing through the point $(-1, 2, -3)$ is perpendicular to the plane P given by $2x + 3y + z + 5 = 0$.

55. What is the equation of the line L ?

- (a) $1 - x = y + 2 = 3 - z$
- (b) $-(x + 1) = y - 2 = z + 3$
- (c) $3x + 3 = 2y - 4 = 6z + 18$
- (d) $3x - 3 = 2y + 4 = 6z - 18$

56. What are the direction ratios of a line M parallel to the plane P ?

- (a) $\langle -3, 2, 1 \rangle$
- (b) $\langle 3, 2, -6 \rangle$
- (c) $\langle 1, 3, 2 \rangle$
- (d) $\langle 2, 2, -10 \rangle$

For the next *two (02)* items that follow :

Let $X(a, p)$, $Y(b, q)$ and $Z(c, r)$ be the points such that a, b and c are in AP.

57. If p, q and r are in AP, then the points X, Y and Z are

- (a) on a straight line
- (b) on a circle
- (c) on a parabola
- (d) the vertices of a triangle

58. If p, q and r are not in AP and $b = c$, then the line joining the points X, Y and Z is parallel to

- (a) y-axis
- (b) x-axis
- (c) $y = x$
- (d) $y = -x$

For the next *two (02)* items that follow :

Consider the points $A(0, 2)$, $B(2, 3)$, $C(4, 5)$ and $D(0, k)$.

59. If the points lie on a circle, then what is/are the possible value(s) of k ?

- (a) 2 only
- (b) 5 only
- (c) 2, 17
- (d) 5, 17

60. If a circle is drawn through A, B and D , then what is the diameter of the circle ?

- (a) $3\sqrt{10}$
- (b) $5\sqrt{10}$
- (c) $3\sqrt{12}$
- (d) $5\sqrt{12}$

For the next *two (02)* items that follow :

The foci of the ellipse $px^2 + 16y^2 = 16p$ and the foci of the hyperbola $25(81x^2 - 144y^2) = 11664$ coincide (assume $p < 16$).

61. What is the value of p ?

- (a) $\sqrt{7}$
- (b) 3
- (c) 7
- (d) 9

62. What is the difference between the eccentricities of the hyperbola and the ellipse ?

- (a) 0.5
- (b) 0.75
- (c) 1.0
- (d) 1.25

For the next *two (02)* items that follow :

The equation of the sphere S is $x^2 + y^2 + z^2 - 4x - 6y - 12z + k = 0$.

63. What is the radius of the sphere passing through origin and concentric with the sphere S ?

- (a) $\frac{7}{2}$
- (b) 5
- (c) 7
- (d) Cannot be determined due to insufficient data

64. If the radius of the sphere S is 8 units, what is the value of k ?

- (a) -15
- (b) 7
- (c) 10
- (d) 15

For the next *two (02)* items that follow :

Let $\vec{a}, \vec{b}, \vec{c}, \vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{a}+\vec{b}+\vec{c}$ be unit vectors.

65. What is the angle between \vec{a} and \vec{b} ?

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) $\frac{2\pi}{3}$

66. What is the angle between \vec{a} and \vec{c} ?

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) $\frac{2\pi}{3}$

For the next two (02) items that follow :

Let $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$

67. Consider the following statements :

- I. $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs.
- II. $\vec{a}, \vec{b}, \vec{c}$ are unit vectors.

Which of the statements given above is/are correct ?

- (a) I only
- ~~(b) II only~~
- (c) Both I and II
- (d) Neither I nor II

68. Consider the following statements :

I. $(\vec{a} \times \vec{b}) \cdot \vec{c} + (\vec{b} \times \vec{c}) \cdot \vec{a} = (\vec{c} \times \vec{a}) \cdot \vec{b}$

II. $\{(\vec{a} \times \vec{b}) \times (\vec{b} \times \vec{c})\} \cdot \vec{b} = 1$

Which of the statements given above is/are correct ?

- ~~(a) I only~~
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

For the next two (02) items that follow :

Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors. Further, \vec{a} is perpendicular to \vec{b} ; \vec{c} makes an angle $\frac{\pi}{3}$ with both \vec{a} and \vec{b} ; and $\vec{c} = p\vec{a} + q\vec{b} + r(\vec{a} \times \vec{b})$.

69. What is the value of $(p + q)$?

- (a) $\frac{1}{2}$
- (b) 1
- (c) $\frac{3}{2}$
- (d) 2

70. What is the value of r^2 ?

- (a) 4
- (b) 2
- (c) 1
- (d) $\frac{1}{2}$

71. If $f(x)$ is differentiable at $x = a$, then consider the following statements :

I. $f(x)$ is continuous at $x = a$

II. $\lim_{x \rightarrow a} f(x) = f(a)$

Which of the statements given above is/are correct ?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

72. What is $\lim_{x \rightarrow 1} \frac{x^{(n^2-1)} - 1}{x^{(n+1)} - 1}$ equal to, where $n > 1$ is a natural number ?

(a) 0

(b) 1

(c) $n - 1$

(d) $n + 1$

73. What is $\lim_{x \rightarrow 0} \frac{10^{\sin x} - 1}{\tan x}$ equal to ?

(a) 0

(b) 1

(c) $\ln 10$

(d) $\log_{10} e$

74. What is the derivative of $\frac{x}{|x|}$ with respect to x , where $x < 0$?

(a) -1

(b) 0

(c) 1

(d) x

75. Consider the following statements in respect of the function $f(x) = x$ in the interval $(-1, 1)$:

I. The function attains maximum value.

II. The function attains minimum value.

Which of the statements given above is/are correct ?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

76. If $\sqrt[3]{y} = x + \sqrt{x^2 + 4}$, then what is

$\sqrt{x^2 + 4} \frac{dy}{dx}$ equal to ?

(a) $\frac{y}{4}$

(b) y

(c) $2y$

(d) $4y$

77. What is the length of the longest interval in which the function $f(x) = 2\cos^2x - 1$ is decreasing?

- (a) 2π
- (b) π
- (c) $\frac{\pi}{2}$
- (d) $\frac{\pi}{4}$

78. If A and B are acute angles such that $2A + 2B = \pi$, then what is the maximum value of $\sin A \cdot \sin B$?

- (a) $\frac{1}{2}$
- (b) $\frac{1}{4}$
- (c) $\frac{\sqrt{3}}{4}$
- (d) 1

79. What is the solution of the differential

equation $\cos\left(\frac{dy}{dx}\right) = p$ when $y(0) = q$?

- (a) $\cos\left(\frac{y-q}{x}\right) = p$
- (b) $\cos\left(\frac{y-p}{x}\right) = q$
- (c) $\cos^{-1}\left(\frac{y-q}{x}\right) = p$
- (d) $\cos^{-1}\left(\frac{y-p}{x}\right) = q$

80. If $2f(x) + f(1-x) = x$, then what is $f(x)$ equal to?

- (a) $x - 1$
- (b) $x - \left(\frac{1}{3}\right)$
- (c) $2x$
- (d) $2x - 1$

For the next two (02) items that follow:

Let $(e^y)^x - y = 0$, where y is a function of x whose domain is $(0, 10]$.

81. What is $\frac{dy}{dx}$ equal to?

- (a) $\frac{y}{1-xy}$
- (b) $\frac{y}{1+xy}$
- (c) $\frac{y^2}{1-xy}$
- (d) $\frac{y^2}{1+xy}$

82. What is $\frac{dy}{dx}$ equal to, given that $y = y_0$ when $x = 1$?

- (a) $-\frac{y_0}{1+e^{y_0}}$
- (b) $-\frac{y_0 e^{y_0}}{1+e^{y_0}}$
- (c) $\frac{y_0 e^{y_0}}{1+e^{y_0}}$
- (d) $\frac{y_0 e^{y_0}}{1-e^{y_0}}$

For the next *two* (02) items that follow :

$$\text{Let } \int_0^{\frac{\pi}{2}} \frac{a \sin x + b \cos x}{(a+b)(\sin x + \cos x)} dx = k$$

83. What is the value of k ?

- (a) $\frac{\pi}{4}$
- (b) $\frac{\pi}{2}$
- (c) π
- (d) 2π

84. What is $\int_0^{\frac{\pi}{2}} \frac{a \cos x + b \sin x}{\sin x + \cos x} dx$ equal to?

- (a) k
- (b) $2k$
- (c) $k(a+b)$
- (d) $\frac{k}{a+b}$

For the next *two* (02) items that follow :

Let S and T be the sets where

$f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 6x + 7$ decreases and increases respectively.

85. What is T equal to?

- (a) $\{x \leq 2\} \cup \{x \geq 3\}$
- (b) $\{x < 2\} \cup \{x > 3\}$
- (c) $(2, 3)$
- (d) $[2, 3]$

86. What is S equal to?

- (a) $\{x \leq 2\} \cup \{x \geq 3\}$
- (b) $\{x < 2\} \cup \{x > 3\}$
- (c) $(2, 3)$
- (d) $[2, 3]$

For the next *two* (02) items that follow :

Let k be the area between the curve $y = \sin x$ and x -axis in the

interval $\left[0, \frac{\pi}{4}\right]$

87. What is the area between the curve $y = \sin x$ and the x -axis in the

interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$?

- (a) k
- (b) $1 - k$
- (c) $\frac{(\pi - k)}{2}$
- (d) $\frac{(\pi - 2k)}{2}$

88. What is the area between the curve $y = \cos x$ and the x -axis in the

interval $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$?

- (a) k
- (b) $1 - k$
- (c) $\frac{(\pi - k)}{2}$
- (d) $\frac{(\pi - 2k)}{2}$

For the next *two* (02) items that follow :

Consider the curves $y = x^2$ and $y = 2|x|$.

89. What is the number of points of intersection of the curves ?

- (a) 4
- (b) 3
- (c) 2
- (d) None

90. What is the area bounded by the curves, the lines $x = 0$ and $x = 1$?

- (a) 1 square unit
- (b) $\frac{2}{3}$ square unit
- (c) $\frac{1}{2}$ square unit
- (d) $\frac{1}{3}$ square unit

$x=2$
 $x=1$

For the next *two* (02) items that follow :

Let

$$\int \frac{\sin \theta d\theta}{(2 + \cos \theta)(3 + 4 \cos \theta)} = A \ln |2 + \cos \theta| + B \ln |3 + 4 \cos \theta|$$

91. What is the value of A ?

- (a) $-\frac{2}{5}$
- (b) $-\frac{1}{5}$

(c) $\frac{1}{5}$

(d) $\frac{2}{5}$

92. What is the value of B ?

(a) $-\frac{2}{5}$

(b) $-\frac{1}{5}$

(c) $\frac{1}{5}$

(d) $\frac{2}{5}$

For the next *two* (02) items that follow :

Let

$$f(x) = \sin x \text{ and } g(x) - f(x) = f(4 - x).$$

93. What is $\int_0^4 \frac{f(x)}{g(x)} dx$ equal to ?

- (a) 0
- (b) 1
- (c) 2
- (d) 4

94. What is $\int_0^4 \frac{f(4-x)}{g(4-x)} dx$ equal to ?

- (a) 0
- (b) 1
- (c) 2
- (d) 4

For the next *two (02)* items that follow :

$$\text{Let } f(x) = \begin{cases} ax(x-1), & x < 1 \\ x-1, & 1 \leq x \leq 3 \\ px^2 + qx + 2, & x > 3 \end{cases}$$

Given that $f(x)$ is continuous for all x but not differentiable at $x = 1$. Further $f'(x)$ is continuous at $x = 3$.

95. What is the value of p ?

(a) -1

(b) $-\frac{1}{3}$

(c) $\frac{1}{3}$

(d) 1

96. What is the value of q ?

(a) -1

(b) $-\frac{1}{3}$

(c) $\frac{1}{3}$

(d) 1

For the next *two (02)* items that follow :

Consider the differential equation

$$e^{x+y} \frac{dy}{dx} = e^{x-y} :$$

97. What is $\frac{d^2y}{dx^2} \left(\frac{dx}{dy} \right)^2$ equal to ?

(a) -2

(b) -1

(c) 0

(d) 2

98. What is the solution of the differential equation with $y(0) = 0$?

(a) $y = \ln(2x + 1)$

(b) $y = \ln(2x - 1)$

(c) $2y = \ln(2x + 1)$

(d) $2y = \ln(2x - 1)$

For the next *two (02)* items that follow :

Let $f(x) = \tan(x^2)$ and

$$g(x) = x|x| \text{ for } |x| < \sqrt{\frac{\pi}{2}}$$

99. If $p(x) = f(x)g(x)$, then which of the following statements is/are correct ?

I. $p(x)$ is continuous at $x = 0$.

II. $p(x)$ is differentiable at $x = 0$.

Select the answer using the code given below :

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

100. If $q(x) = f \circ g(x)$, then which of the following statements is/are correct?

- I. $q(x)$ is continuous at $x = 0$.
- II. $q(x)$ is differentiable at $x = 0$.

Select the answer using the code given below :

- (a) I only
- (b) II only
- (c) Both I and II
- (d) Neither I nor II

For the next *five (05)* items that follow :

A 4-digit number is selected at random formed by using the digits 0, 1, 2, 3 and 4 (where repetition of digits is not allowed).

101. What is the probability that the number selected is divisible by 2?

- (a) $\frac{5}{8}$
- (b) $\frac{3}{8}$
- (c) $\frac{1}{8}$
- (d) $\frac{5}{24}$

Handwritten notes for question 101: $18 \frac{1}{2}$, 4×2 , a table with numbers 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100, and $2(2)$.

102. What is the probability that the number selected is divisible by 3?

- (a) $\frac{9}{28}$

(b) $\frac{3}{8}$

(c) $\frac{3}{16}$

(d) $\frac{8}{25}$

103. What is the probability that the number selected is divisible by 4?

(a) $\frac{1}{2}$

(b) $\frac{7}{16}$

(c) $\frac{9}{16}$

(d) $\frac{5}{16}$

104. What is the probability that the number selected is divisible by 6?

(a) $\frac{7}{48}$

(b) $\frac{2}{3}$

(c) $\frac{1}{4}$

(d) $\frac{3}{8}$

105. What is the probability that the number selected does not contain zero at any position?

- (a) 0.12
- (b) 0.25
- (c) 0.375
- (d) 0.45

1, 2, 3, 4

(c) $\frac{4}{21}$

(d) $\frac{5}{28}$

For the next *three (03)* items that follow:

Let A, B, C and D be mutually exclusive and exhaustive events and

$$\frac{P(A)}{2} = \frac{P(B)}{3} = \frac{P(C)}{5} = \frac{P(D)}{8}$$

106. What is $P(A) + P(B) + P(C)$ equal to?

(a) $\frac{5}{9}$

(b) $\frac{1}{18}$

(c) $\frac{2}{21}$

(d) $\frac{7}{13}$

107. What is $\frac{[2P(A) + 3P(B)]}{[4P(C) + 5P(D)]}$ equal to?

(a) $\frac{13}{18}$

(b) $\frac{13}{60}$

105-c

108. If G is the geometric mean of P(A), P(B), P(C) and P(D), then what is 9G equal to?

(a) $17^{\frac{1}{4}}$

(b) $15^{\frac{1}{4}}$

(c) $13^{\frac{1}{4}}$

(d) $11^{\frac{1}{4}}$

Handwritten notes: u, G, p, q, a, b

For the next *two (02)* items that follow:

Four dice are rolled.

109. What is the probability that getting a total of the numbers on the dice is 6?

(a) $\frac{1}{216}$

(b) $\frac{1}{324}$

(c) $\frac{5}{648}$

(d) $\frac{7}{648}$

Handwritten note: $1, 2, 3, 4, 5, 6$

Handwritten note: $4/2$

Handwritten note: $6 \times 6 \times 6 \times 6$

112-a

110. What is the probability that getting a total of the numbers on the dice is at least 23 ?

(a) $\frac{1}{1296}$

(b) $\frac{1}{432}$

(c) $\frac{1}{324}$

(d) $\frac{5}{1296}$

Handwritten notes for Q110: $6 \times 6 \times 6 \times 6$, $(5,1), (1,2)$, $(1,1,1), (1,2,3)$, $(1,2,1)$, $(6,6,6)$

111. Let two lines of regression be $x + y + 11 = 0$ and $2x + 3y + 4 = 0$ for some data. What is the value of correlation coefficient between x and y ?

(a) $-\sqrt{\frac{2}{3}}$

(b) $-\sqrt{\frac{1}{6}}$

(c) $\sqrt{\frac{2}{3}}$

(d) $\sqrt{\frac{1}{6}}$

112. If the mean of 20 observations, namely $x_1, x_2, x_3, \dots, x_{20}$ is 1.414, then what is the value of $\sum_{i=1}^{20} 100(2x_i + 4)$?

(a) 24168

(b) 20828

(c) 15248

(d) 13656

Handwritten notes for Q112: 1.414 , $100 \times 1.414 = \sum x_i$

113. In an entrance test there are multiple choice questions. There are four options for each question, of which only one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then what is the probability that he was guessing ?

(a) $\frac{37}{40}$

(b) $\frac{36}{37}$

(c) $\frac{1}{37}$

(d) $\frac{1}{40}$

Handwritten notes for Q113: 90% , $1 - \frac{90}{100} = \frac{10}{100}$, $\frac{10}{100} \times \frac{1}{4} = \frac{1}{40}$, $1 - \frac{1}{40} = \frac{39}{40}$, $\frac{39}{40} \times \frac{1}{37} = \frac{39}{1480}$, $\frac{39}{1480} \times 100 = \frac{3900}{1480} = \frac{37}{40}$

114. Consider the following statements in respect of the events A, B, C :

I. $(A \cup B \cup C) \cap (\bar{A} \cap \bar{B} \cap \bar{C})$ is an impossible event.

II. $(A \cap B \cap C) \cap (\bar{A} \cup \bar{B} \cup \bar{C})$ is a possible event.

Which of the statements given above is/are correct ?

(a) I only

(b) II only

(c) Both I and II

(d) Neither I nor II

Handwritten note: $\frac{39}{1480}$

115. The standard deviation of 100 observations is 10. If 5 is multiplied to each of the observations, then what is the new standard deviation?

- (a) 20
- (b) 25
- (c) 40
- ✓(d) 50

116. For a Binomial distribution with mean 6 and standard deviation $\sqrt{2}$, what is the value of $P(X=0)$?

- (a) $\left(\frac{1}{3}\right)^9$
- (b) $\left(\frac{2}{3}\right)^9$
- (c) $\frac{1}{3}\left(\frac{2}{3}\right)^8$
- (d) $\frac{2}{3}\left(\frac{1}{3}\right)^8$

$n=9$
 $p=2/3$
 $q=1/3$
 $Y=2X-5$

117. If the random variable X has mean 3 and standard deviation 5, then what is the variance of the random variable $Y=2X-5$?

- (a) 15
- (b) 40
- (c) 45
- (d) 100

$\bar{x} = 3$
 $\sigma = 5$

118. Three events A, B and C are such that A and B are disjoint, A and C are independent, B and C are independent. If $4P(A) = 2P(B) = P(C)$ and $P(A \cup B \cup C) = 5P(A)$, then what is the value of $P(C)$?

- (a) $\frac{5}{6}$
- (b) $\frac{1}{3}$
- (c) $\frac{1}{6}$
- (d) $\frac{2}{3}$

For the next *two (02)* items that follow:

The frequency distribution of marks obtained by 100 students in a certain subject is given below:

Marks	0-10	10-20	20-30	30-40
No. of students	10	20	f	40

$f=30$
 $=100$
 $=30$
 $n=25$

119. What is the mean deviation about the arithmetic mean?

- (a) 5.5
- (b) 6
- (c) 6.5
- (d) 8

120. What is the standard deviation?

- (a) 8.5
- (b) 9
- (c) 9.5
- (d) 10