

NDA MATHEMATICS — SET A (NPSS-A-HMT)

COMPLETE SOLUTIONS — 120 Questions

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ALGEBRA, SEQUENCES & SERIES (Q1-Q10)

Q1. p, q, r in AP. $(q-p), (r-q), p$ in GP. Find $(p+q):(q+r):(r+p)$

Common difference = d . So $q = p+d, r = p+2d$
 $(q-p) = d, (r-q) = d, p$ in GP $\Rightarrow d^2 = d \cdot p \Rightarrow d = p$
 $q = 2p, r = 3p$
 $(p+q):(q+r):(r+p) = 3p:5p:4p = 3:5:4$

Answer: (c) 3 : 5 : 4

Q2. p, g_1, g_2, q in GP. m = AM of p, q . Find $g_1^2/g_2 + g_2^2/g_1$

Let common ratio = r . Then $g_1 = pr, g_2 = pr^2$
 $g_1^2/g_2 = p^2r^2/(pr^2) = p$
 $g_2^2/g_1 = p^2r^4/(pr) = pr^3 = q$
Sum = $p + q = 2m$

Answer: (b) 2m

Q3. Complex number inequalities: I. $1+4i > 3+2i$, II. $2+3i < 3+4i$, III. $4+3i > 3+4i$. How many valid?

Complex numbers cannot be compared using $<$ or $>$ (no total ordering on \mathbb{C}).
None of the three inequalities are valid.

Answer: (a) None

Q4. $3Z_1/(4Z_2)$ is purely imaginary. Find $|Z_1+Z_2|/|Z_1-Z_2|$

Let $Z_1/Z_2 = ti$ (purely imaginary, absorbing constants)
 $|Z_1+Z_2|/|Z_1-Z_2| = |ti+1|/|ti-1| = \sqrt{(1+t^2)}/\sqrt{(1+t^2)} = 1$

Answer: (d) 1

Q5. α, β, γ are cube roots of -8 . Find $(\alpha^2p^2+\beta^2q^2+\gamma^2r^2)/(\beta^2p^2+\gamma^2q^2+\alpha^2r^2)$

Cube roots of -8 : $\alpha = -2, \beta = -2\omega, \gamma = -2\omega^2$
 $\alpha^2 = 4, \beta^2 = 4\omega^2, \gamma^2 = 4\omega$
Numerator = $4(p^2 + \omega^2q^2 + \omega r^2)$, Denominator = $4(\omega^2p^2 + \omega q^2 + r^2)$
Multiply num/den by ω : ratio = $\omega = \gamma/\beta$

Answer: (b) γ/β

Q6. $S_n = 3n^2 + 5n$. If m^{th} term = 68, find m .

$a_m = S_m - S_{m-1} = (3m^2+5m) - (3(m-1)^2+5(m-1))$
 $= 3m^2+5m - 3m^2+6m-3 - 5m+5 = 6m + 2$
 $6m + 2 = 68 \Rightarrow 6m = 66 \Rightarrow m = 11$

Answer: (c) 11

Q7. Set S has $(2n+1)$ elements. Subsets with at most n elements = 1024. Find n .

Total subsets = 2^{2n+1} . By symmetry (subset \leftrightarrow complement):
Subsets with $\leq n$ elements = $2^{2n+1}/2 = 2^{2n}$
 $2^{2n} = 1024 = 2^{10} \Rightarrow 2n = 10 \Rightarrow n = 5$

Answer: (d) 5

Q8. Maximum intersection points of 5 circles?

Each pair of circles intersects in at most 2 points.

Pairs = ${}^5C_2 = 10$. Maximum = $10 \times 2 = 20$

Answer: (c) 20

Q9. Greatest value of r: ${}^{15}C_{r+1} > 2 \times {}^{15}C_r$

$${}^{15}C_{r+1}/{}^{15}C_r = (15-r)/(r+1) > 2$$

$$15 - r > 2r + 2 \Rightarrow 13 > 3r \Rightarrow r < 4.33$$

Greatest integer $r = 4$

Answer: (c) 4

Q10. If $n = {}^mC_2$, then ${}^nC_2 = ?$

$$n = m(m-1)/2$$

$${}^nC_2 = n(n-1)/2 = [m(m-1)/2] \times [m(m-1)/2 - 1] / 2$$

$$= m(m-1)(m^2-m-2)/8 = m(m-1)(m+1)(m-2)/8$$

$$= 3 \times (m+1)m(m-1)(m-2)/24 = 3 \times {}^{m+1}C_4$$

Answer: (c) $3 \times {}^{m+1}C_4$

MATRICES & DETERMINANTS (Q11-Q22)**Q11. Quadratic equations (leading coeff = 1) unchanged on squaring roots. Total number?**

Case 1: $r^2=r, s^2=s \Rightarrow r, s \in \{0,1\}$. Quadratics: $x^2, x^2-x, (x-1)^2 \rightarrow 3$ equations

Case 2: $r^2=s, s^2=r, r \neq s \Rightarrow r^3=1 \Rightarrow r=\omega, \omega^2 \rightarrow x^2+x+1 \rightarrow 1$ equation

Total = $3 + 1 = 4$

Answer: (b) 4

Q12. Roots of $x^2-2bx+c^2=0$ are α, β . $A=AM, G=GM$. Roots of $x^2-(b+c)x+bc=0$?

$$A = (\alpha+\beta)/2 = 2b/2 = b$$

$$G = \sqrt{\alpha\beta} = \sqrt{c^2} = c$$

$$x^2-(b+c)x+bc = (x-b)(x-c) = (x-A)(x-G). \text{ Roots} = A, G$$

Answer: (a) A, G

Q13. If $1 - \log_{10} 2 = \log_{10}(5^x + 4^x + 3^x + 2^x + 1)$, find x.

$$1 - \log_{10} 2 = \log_{10}(10/2) = \log_{10} 5$$

$$\text{So } 5 = 5^x + 4^x + 3^x + 2^x + 1$$

$$\text{At } x = 0: 1+1+1+1+1 = 5 \checkmark$$

Answer: (d) 0

Q14. $f(x) = |3x^2, \cos x, -\sin x; 6, -1, 0; q, q^2, q^3|$. Find d^2f/dx^2 at $x=0$.

Expanding along R1: $f(x) = -3q^3x^2 - 6q^3\cos x - q(6q+1)\sin x$

$$f(x) = -6q^3x + 6q^3\sin x - q(6q+1)\cos x$$

$$f'(x) = -6q^3 + 6q^3\cos x + q(6q+1)\sin x$$

$$f'(0) = -6q^3 + 6q^3 \cdot 1 + 0 = 0$$

Answer: (b) 0

Q15. $|a-b, p-q, x-y; b-c, q-r, y-z; c-a, r-p, z-x| = k|a,b,c; p,q,r; x,y,z|$. Find k.

$R_3 = -(R_1 + R_2)$ since $(c-a) = -(a-b) - (b-c)$, etc.

When a row is linear combination of others, determinant = 0. So LHS = 0 always.

Therefore $k = 0$

Answer: (b) 0

Q16. p, q, r are cube roots of unity. Find $|p^2+q^2, r^2, r^2; p^2, q^2+r^2, p^2; q^2, q^2, r^2+p^2|$

$p=1, q=\omega, r=\omega^2$. So $p^2=1, q^2=\omega^2, r^2=\omega$ (since $\omega^4=\omega$)

Using $1+\omega+\omega^2=0$: Matrix becomes $[-\omega, \omega, \omega; 1, -1, 1; \omega^2, \omega^2, -\omega^2]$

Expanding: $= 0 + 2\omega^3 + 2\omega^3 = 4(1) = 4$

Answer: (d) 4

Q17. $|A| = -2$. Find $|AA^T|$.

$|AA^T| = |A| \cdot |A^T| = |A|^2 = (-2)^2 = 4$

Answer: (d) 4

Q18. I: Symmetric matrix \Rightarrow inverse is symmetric. II: Singular matrix \Rightarrow adjoint is singular.

I: $(A^{-1})^T = (A^T)^{-1} = A^{-1}$ (since $A=A^T$). \checkmark

II: $|\text{adj}(A)| = |A|^{n-1} = 0^{n-1} = 0$ for $n > 1$. So $\text{adj}(A)$ is singular. \checkmark

Answer: (c) Both I and II

Q19. $M = \text{diag}(2,2,2)$. Find $|M| \cdot |\text{adj } M|$.

$|M| = 2^3 = 8$

$|\text{adj } M| = |M|^{n-1} = 8^2 = 64$

$|M| \cdot |\text{adj } M| = 8 \times 64 = 512$

Answer: (d) 512

Q20. $M_k = [[k, k-1], [k-1, k]]$. Find $\sum |M_k|$ for $k=1$ to 50 .

$|M_k| = k^2 - (k-1)^2 = 2k - 1$

Sum $= 1+3+5+\dots+99 = 50^2 = 2500$

Answer: (c) 2500

Q21. 2×2 matrix: I. $|M+M^T| = |M| + |M^T|$ if symmetric. II. $|M+M^T| = 0$ if anti-symmetric.

I: $|2M| = 4|M| \neq 2|M|$ in general. \times

II: $M^T = -M \Rightarrow M+M^T = 0 \Rightarrow |0| = 0$. \checkmark

Answer: (b) II only

Q22. $M^3 = M$. How many values of $|M|$ possible?

$|M|^3 = |M| \Rightarrow |M|(|M|^2 - 1) = 0 \Rightarrow |M| = 0, 1, \text{ or } -1$. Three values.

Answer: (c) Three

SETS, RELATIONS & BOOLEAN (Q23-Q25)

Q23. $p=(x+y+z), q=xyz$. Det $|x,1,1;1,y,1;1,1,z| > 0$. Which is correct?

Det $= xyz - x - y - z + 2 = q - p + 2$

Det $> 0 \Rightarrow q + 2 > p$

Answer: (c) $q + 2 > p$

Q24. $((A \cap B) \cup (A - B)) - ((A \cap B) \cup (B - A)) \cup A = ?$

$(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B^c) = A$

$(A \cap B) \cup (B - A) = (A \cap B) \cup (B \cap A^c) = B$

Result = $(A - B) \cup A = A$

Answer: (b) A

Q25. $A \cap C = B \cap C = \emptyset$, $A \cup C = B \cup C$. Which is true: I. $C = \emptyset$, II. $A = B$, III. $A \cup B = C$?

$x \in A \Rightarrow x \notin C \Rightarrow x \in A \cup C = B \cup C \Rightarrow x \in B$. So $A \subseteq B$. Similarly $B \subseteq A$.

Hence $A = B$. C need not be \emptyset . $A \cup B \neq C$ necessarily.

Answer: (b) II only

TRIGONOMETRY (Q26-Q40)

Q26. $2 \sec 4\beta = \tan 2\alpha + \cot 2\alpha$. Find $(\alpha + \beta)$.

$\tan 2\alpha + \cot 2\alpha = (\sin^2 2\alpha + \cos^2 2\alpha)/(\sin 2\alpha \cdot \cos 2\alpha) = 2/\sin 4\alpha$

$2 \sec 4\beta = 2/\sin 4\alpha \Rightarrow \sin 4\alpha = \cos 4\beta$

$4\alpha = \pi/2 - 4\beta \Rightarrow \alpha + \beta = \pi/8$

Answer: (d) $\pi/8$

Q27. $\alpha + \beta = \pi/2$, $\alpha - \beta = \pi/6$, $m \tan \beta = n \tan \alpha$. Find $(m+n)/(m-n)$.

$\alpha = \pi/3$, $\beta = \pi/6$

$m \cdot \tan(\pi/6) = n \cdot \tan(\pi/3) \Rightarrow m/\sqrt{3} = n\sqrt{3} \Rightarrow m = 3n$

$(m+n)/(m-n) = 4n/2n = 2$

Answer: (a) 2

Q28. $x = \sec \theta - \tan \theta$, $y = \csc \theta + \cot \theta$. Which relation holds?

Test $\theta = \pi/4$: $x = \sqrt{2}-1$, $y = \sqrt{2}+1$, $xy = 1$

(b): $x-y+xy+1 = (\sqrt{2}-1)-(\sqrt{2}+1)+1+1 = -2+2 = 0$ ✓

Answer: (b) $x - y + xy + 1 = 0$

Q29. $\cos \theta = 1/3$. Find $\sin(\theta/2) \cdot \sin(3\theta/2)$.

$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

$\sin(\theta/2) \cdot \sin(3\theta/2) = [\cos(-\theta) - \cos(2\theta)]/2 = [\cos \theta - \cos 2\theta]/2$

$\cos 2\theta = 2(1/3)^2 - 1 = 2/9 - 1 = -7/9$

$= [1/3 - (-7/9)]/2 = [1/3 + 7/9]/2 = (10/9)/2 = 5/9$

Answer: (a) 5/9

Q30. $\cos x + \sqrt{3} \sin x$ is maximum when $x = ?$

$\cos x + \sqrt{3} \sin x = 2 \cos(x - \pi/3)$. Maximum when $x = \pi/3$.

Verify: $\cos(\pi/3) + \sqrt{3} \cdot \sin(\pi/3) = 1/2 + 3/2 = 2$ ✓

Answer: (b) $\pi/3$

Q31. θ in Q4, $3 \cot \theta + 4 = 0$. Find $\sin 2\theta + \cos 2\theta$.

$\cot \theta = -4/3$, $\tan \theta = -3/4$. In Q4: $\sin \theta = -3/5$, $\cos \theta = 4/5$

$\sin 2\theta = 2(-3/5)(4/5) = -24/25$

$\cos 2\theta = (16-9)/25 = 7/25$

Sum = $-24/25 + 7/25 = -17/25$

Answer: (b) $-17/25$

Q32. $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$, $\alpha \neq \beta$. Find $\cos 2\alpha + \cos 2\beta + 2 \cos(\alpha + \beta)$.

$\cos \alpha = -\cos \beta$, $\sin \alpha = -\sin \beta \Rightarrow \alpha = \beta + \pi$

$\cos 2\alpha = \cos(2\beta + 2\pi) = \cos 2\beta$

$$\cos(\alpha+\beta) = \cos(2\beta+\pi) = -\cos 2\beta$$

$$\text{Sum} = 2 \cos 2\beta + 2(-\cos 2\beta) = 0$$

Answer: (a) 0

Q33. In $\triangle ABC$, $\sin A = \cos B + \cos C$. Find $\tan(B/2) + \cot(B/2)$.

$$\sin A = 2 \cos((B+C)/2) \cos((B-C)/2) \text{ and } \cos((B+C)/2) = \sin(A/2)$$

$$2 \sin(A/2) \cos(A/2) = 2 \sin(A/2) \cos((B-C)/2)$$

$$\Rightarrow \cos(A/2) = \cos((B-C)/2) \Rightarrow A = B-C$$

$$A+B+C = \pi \Rightarrow 2B = \pi \Rightarrow B = \pi/2$$

$$\tan(\pi/4) + \cot(\pi/4) = 1 + 1 = 2$$

Answer: (d) 2

Q34. Inverse circular functions: I. $\sin^{-1}(-x) = -\sin^{-1}x$, II. $\cos^{-1}(-x) = \cos^{-1}x$, III. $\tan^{-1}(-x) = \pi - \tan^{-1}x$, IV. $\cot^{-1}(-x) = \pi - \cot^{-1}x$. How many correct?

- I. \checkmark (odd function property)
 II. \times (correct: $\cos^{-1}(-x) = \pi - \cos^{-1}x$)
 III. \times (correct: $\tan^{-1}(-x) = -\tan^{-1}x$)
 IV. \checkmark (standard identity)
 Two are correct (I and IV).

Answer: (b) Two

Q35. Find $\tan[2 \tan^{-1}(1/2) - \pi/4]$.

$$\text{Let } \phi = 2 \tan^{-1}(1/2). \text{ Then } \tan \phi = 2 \cdot (1/2) / (1 - 1/4) = 1 / (3/4) = 4/3$$

$$\tan(\phi - \pi/4) = (\tan \phi - 1) / (1 + \tan \phi) = (4/3 - 1) / (1 + 4/3) = (1/3) / (7/3) = 1/7$$

Answer: (d) 1/7

Q36. $\triangle ABC$ angles 1:1:4. Longest side = 3. Find perimeter.

$$\text{Angles: } 30^\circ, 30^\circ, 120^\circ. \text{ Side opposite } 120^\circ = c = 3.$$

$$a/\sin 30^\circ = 3/\sin 120^\circ \Rightarrow a = 3 \times (1/2) / (\sqrt{3}/2) = 3/\sqrt{3} = \sqrt{3}$$

$$\text{Perimeter} = 3 + \sqrt{3} + \sqrt{3} = 3 + 2\sqrt{3}$$

Answer: (b) $3 + 2\sqrt{3}$ units

Q37. In $\triangle ABC$, find $\sin(A-B)/\sin(A+B)$.

$$a^2 - b^2 = 4R^2(\sin^2 A - \sin^2 B) = 4R^2 \sin(A+B) \sin(A-B)$$

$$c^2 = 4R^2 \sin^2 C = 4R^2 \sin^2(A+B)$$

$$\sin(A-B)/\sin(A+B) = (a^2 - b^2)/c^2$$

Answer: (c) $(a^2 - b^2)/c^2$

Q38. Smallest positive x : $\log_{\sin x}(\cos x) + \log_{\cos x}(\sin x) = 2$

$$\text{Let } a = \ln(\sin x), b = \ln(\cos x). \text{ Then } b/a + a/b = 2$$

$$(a^2 + b^2)/(ab) = 2 \Rightarrow (a-b)^2 = 0 \Rightarrow a = b$$

$$\sin x = \cos x \Rightarrow x = \pi/4$$

Answer: (c) $\pi/4$

Q39. Plane at 10 km, elevation 67.5° . Height?

$$\text{Height} = 10 \sin(67.5^\circ) = 10 \cos(22.5^\circ)$$

$$\cos(22.5^\circ) = \sqrt{(1 + \cos 45^\circ)/2} = \sqrt{(2 + \sqrt{2})/4} = \sqrt{(2 + \sqrt{2})}/2$$

$$\text{Height} = 10 \times \sqrt{(2 + \sqrt{2})}/2 = 5\sqrt{(2 + \sqrt{2})}$$

Answer: (c) $5\sqrt{(2 + \sqrt{2})}$ km

Q40. Chord of unit circle subtending 45° at centre.

$$\text{Chord} = 2r \sin(\theta/2) = 2(1) \sin(22.5^\circ)$$

$$\sin(22.5^\circ) = \sqrt{(1 - \cos 45^\circ)/2} = \sqrt{(2 - \sqrt{2})/4} = \sqrt{2 - \sqrt{2}}/2$$

$$\text{Chord} = \sqrt{2 - \sqrt{2}}$$

Answer: (d) $\sqrt{2 - \sqrt{2}}$ units

RELATIONS & CALCULUS — BASICS (Q41-Q50)**Q41. $R = \{(x, y) : x, y \in \mathbb{N}, x = y^3\}$. Symmetric? Transitive?**

Symmetric: $(8, 2) \in R$ ($8 = 2^3$) but $(2, 8) \notin R$ ($2 \neq 8^3$). \times

Transitive: $(512, 8) \in R$, $(8, 2) \in R$, but $(512, 2) \notin R$ ($512 \neq 2^3 = 8$). \times

Answer: (d) Neither I nor II

Q42. Minimum value of $x^2 + kx + k^2$

$$\text{Min at } x = -k/2: f(-k/2) = k^2/4 - k^2/2 + k^2 = 3k^2/4$$

Answer: (c) $3k^2/4$

Q43. I: $\sqrt{x+x+1}=0$ has two irrational roots. II: $5\sqrt{x-x-4}=0$ has two rational roots.

I: $\sqrt{x} \geq 0$ but $\sqrt{x} = -(x+1)$ needs $x \leq -1$, contradicts $x \geq 0$. No solution. \times

II: Let $t = \sqrt{x}$: $t^2 - 5t + 4 = 0 \Rightarrow t = 1, 4 \Rightarrow x = 1, 16$ (both rational). \checkmark

Answer: (b) II only

Q44. Numbers > 1000 from $\{0, 1, 2, 3\}$, no repetition.

4-digit numbers: 1st digit $\neq 0 \Rightarrow 3$ choices. Rest: $3 \times 2 \times 1 = 6$. Total = 18

Answer: (b) 18

Q45. p^{th} term of AP = k . Sum of p^{th} , $(p+q)^{\text{th}}$, $(p-q)^{\text{th}}$ terms?

$$T_p = k, T_{p+q} = k + qd, T_{p-q} = k - qd. \text{ Sum} = 3k$$

Answer: (b) $3k$

Q46. $(\sqrt{2+1})^{10} = u+f$, $(\sqrt{2-1})^{10} = v$. I: $(u+v+f)$ is integer. II: $(f+v)$ is integer.

By binomial theorem: $(\sqrt{2+1})^{10} + (\sqrt{2-1})^{10} = \text{integer}$ (irrational parts cancel)

So $(u+f)+v = \text{integer} \Rightarrow \text{I} \checkmark$

u is integer, so $f+v$ must be integer. Since $0 < f < 1$, $0 < v < 1$: $f+v=1 \Rightarrow \text{II} \checkmark$

Answer: (c) Both I and II

Q47. Multiplicative inverse of $(\sqrt{2+1})^{20}$?

$$(\sqrt{2+1})(\sqrt{2-1}) = 1 \Rightarrow (\sqrt{2+1})^{20} \cdot (\sqrt{2-1})^{20} = 1$$

$$\text{Inverse} = (\sqrt{2-1})^{20} = [(\sqrt{2-1})^{10}]^2 = v^2$$

Answer: (c) v^2

Q48. Value of $(v+f)$?

From Q46: $f+v = 1$

Answer: (b) 1

Q49. Value of u ?

$$u = (\sqrt{2}+1)^{10} + (\sqrt{2}-1)^{10} - 1 \text{ (since } u = \text{integer part, } f+v = 1)$$

$$\text{Using recurrence } S_n = 2\sqrt{2} \cdot S_{n-1} - S_{n-2}:$$

$$S_0=2, S_2=6, S_4=34, S_6=198, S_8=1154, S_{10}=6726$$

$$u = 6726 - 1 = 6725$$

Answer: (d) 6725

Q50. Value of uv ?

$$(u+f)v = (\sqrt{2}+1)^{10} \cdot (\sqrt{2}-1)^{10} = 1$$

$$uv = 1 - fv. \text{ Since } f=1-v: fv = v-v^2 > 0$$

$$\text{So } uv = 1 - (v-v^2) < 1 \text{ and } uv > 0. \text{ Hence } 0 < uv < 1$$

Answer: (c) $0 < uv < 1$

COORDINATE GEOMETRY & 3D (Q51-Q70)

Q51. Line $x - \sqrt{3}y = 0$ makes angle α with L: $x + \sqrt{3}y + 3\sqrt{3} = 0$. Find α .

$$\text{Slope of } x - \sqrt{3}y = 0: m_1 = 1/\sqrt{3} \text{ (angle } 30^\circ)$$

$$\text{Slope of L: } m_2 = -1/\sqrt{3} \text{ (angle } 150^\circ). \text{ Angle} = |150^\circ - 30^\circ| = 120^\circ \text{ or } 60^\circ$$

$$\text{Since } 0 \leq \alpha \leq 90^\circ: \alpha = 60^\circ$$

Answer: (c) 60°

Q52. Angle of L with positive y-axis?

$$\text{Acute angle of L with x-axis} = 30^\circ. \text{ Angle with y-axis} = 90^\circ - 30^\circ = 60^\circ$$

Answer: (c) 60°

Q53. P(-2, -3, 5), Q(4, -1, 5), R(6, -4, 8), S(2, -6, 10). I: PQ||RS. II: PR⊥QS.

$$PQ = (6, 2, 0), RS = (-4, -2, 2). \text{ Ratios } 6/(-4) \neq 2/(-2). \text{ Not parallel. } \times$$

$$PR = (8, -1, 3), QS = (-2, -5, 5). PR \cdot QS = -16 + 5 + 15 = 4 \neq 0. \times$$

Answer: (d) Neither I nor II

Q54. $PQ^2 + 2QS^2 - 2PR^2 = ?$

$$PQ^2=40, QS^2=54, PR^2=74. \text{ Result} = 40+108-148 = 0$$

Answer: (b) 0

Q55. Line through (-1, 2, -3), perpendicular to plane $2x + 3y + z + 5 = 0$. Equation?

$$\text{Direction} = (2, 3, 1). \text{ Line: } (x+1)/2 = (y-2)/3 = (z+3)/1$$

$$\text{Option (c): } 3(x+1) = 2(y-2) = 6(z+3) \div \text{by } 6 \Rightarrow (x+1)/2 = (y-2)/3 = (z+3)/1 \checkmark$$

Answer: (c) $3x+3 = 2y-4 = 6z+18$

Q56. Direction ratios of line parallel to plane $2x + 3y + z + 5 = 0$?

$$\text{Need } 2a+3b+c = 0. \text{ Check (d): } 2(2)+3(2)+(-10) = 4+6-10 = 0 \checkmark$$

Answer: (d) (2, 2, -10)

Q57. X(a, p), Y(b, q), Z(c, r) with a, b, c in AP. If p, q, r also in AP?

$$Y = \text{midpoint of X and Z} \Rightarrow X, Y, Z \text{ are collinear.}$$

Answer: (a) On a straight line

Q58. p, q, r not in AP and b=c. Line XYZ parallel to?

a,b,c in AP with b=c \Rightarrow a=b=c. All x-coordinates equal \Rightarrow parallel to y-axis.

Answer: (a) y-axis

Q59. A(0,2), B(2,3), C(4,5), D(0,k) on a circle. Find k.

Circle: $x^2+y^2+Dx+Ey+F=0$. Using A,B,C: $D=5, E=-19, F=34$

D on circle: $k^2-19k+34=0 \Rightarrow k = (19 \pm 15)/2 \Rightarrow k = 17$ or 2

Answer: (c) 2, 17

Q60. Circle through A, B, D(0,17). Diameter?

Centre $(-5/2, 19/2)$. $r^2 = 25/4 + 361/4 - 34 = 125/2$

Diameter = $2\sqrt{(125/2)} = 5\sqrt{10}$

Answer: (b) $5\sqrt{10}$

Q61. Ellipse $px^2+16y^2=16p$, hyperbola $25(81x^2-144y^2)=11664$. Foci coincide. Find p.

Ellipse foci: $c = \sqrt{16-p}$. Hyperbola: $a^2=144/25, b^2=81/25, c^2=9 \Rightarrow c=3$

$\sqrt{16-p} = 3 \Rightarrow p = 7$

Answer: (c) 7

Q62. Difference of eccentricities of hyperbola and ellipse?

$e_{\text{ellipse}} = 3/4, e_{\text{hyperbola}} = 3/(12/5) = 5/4$. Difference = $5/4 - 3/4 = 0.5$

Answer: (a) 0.5

Q63. Sphere $x^2+y^2+z^2-4x-6y-12z+k=0$. Radius through origin, concentric?

Centre $(2,3,6)$. Distance to O = $\sqrt{4+9+36} = \sqrt{49} = 7$

Answer: (c) 7

Q64. Radius of S = 8 units. Find k.

$\sqrt{4+9+36-k} = 8 \Rightarrow 49-k = 64 \Rightarrow k = -15$

Answer: (a) -15

Q65. $\vec{a}, \vec{b}, \vec{c}, \vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{a}+\vec{b}+\vec{c}$ all unit vectors. Angle between \vec{a} and \vec{b} ?

$|\vec{a}+\vec{b}|^2=1: 1+2(\vec{a}\cdot\vec{b})+1=1 \Rightarrow \vec{a}\cdot\vec{b}=-1/2 \Rightarrow \theta=2\pi/3$

Answer: (d) $2\pi/3$

Q66. Angle between \vec{a} and \vec{c} ?

$|\vec{b}+\vec{c}|^2=1 \Rightarrow \vec{b}\cdot\vec{c}=-1/2$

$|\vec{a}+\vec{b}+\vec{c}|^2=1: 3+2(-1/2-1/2+\vec{a}\cdot\vec{c})=1 \Rightarrow \vec{a}\cdot\vec{c}=0 \Rightarrow \text{angle} = \pi/2$

Answer: (c) $\pi/2$

Q67. $\vec{a}\times\vec{b}=\vec{c}, \vec{b}\times\vec{c}=\vec{a}$. I: Orthogonal in pairs. II: Unit vectors.

I: Cross products ensure mutual perpendicularity. \checkmark

II: $|\vec{b}|=1$ forced, but $|\vec{a}|=|\vec{c}|=r$ (any r). Not necessarily 1. \times

Answer: (a) I only

Q68. Scalar triple product statements with $\vec{a}\times\vec{b}=\vec{c}, \vec{b}\times\vec{c}=\vec{a}$.

I: LHS = $|\vec{c}|^2+|\vec{a}|^2 = 2r^2$. RHS = $[\vec{a},\vec{b},\vec{c}] = r^2$. Not equal. \times

II: Value = r^2 . Equals 1 only if $r=1$ (not forced). X

Answer: (d) Neither I nor II

Q69. Unit vectors $\vec{a} \perp \vec{b}$, \vec{c} makes $\pi/3$ with both. $\vec{c} = p\vec{a} + q\vec{b} + r(\vec{a} \times \vec{b})$. Find $p+q$.

$$\vec{c} \cdot \vec{a} = p = \cos(\pi/3) = 1/2$$

$$\vec{c} \cdot \vec{b} = q = \cos(\pi/3) = 1/2$$

$$p + q = 1$$

Answer: (b) 1

Q70. Find r^2 .

$$|\vec{c}|^2 = p^2 + q^2 + r^2 |\vec{a} \times \vec{b}|^2 = 1/4 + 1/4 + r^2 = 1 \Rightarrow r^2 = 1/2$$

Answer: (d) 1/2

CALCULUS — LIMITS, DERIVATIVES, APPLICATIONS (Q71-Q80)

Q71. $f(x)$ differentiable at $x=a$. I: f continuous at a . II: $\lim_{x \rightarrow a} f(x) = f(a)$.

Differentiability \Rightarrow Continuity $\Rightarrow \lim_{x \rightarrow a} f(x) = f(a)$. Both true.

Answer: (c) Both I and II

Q72. $\lim_{x \rightarrow 1} (x^{n^2-1} - 1)/(x^{n+1} - 1)$, $n > 1$

$$= (n^2 - 1)/(n + 1) = (n - 1)(n + 1)/(n + 1) = n - 1$$

Answer: (c) $n - 1$

Q73. $\lim_{x \rightarrow 0} (10^{\sin x} - 1)/\tan x$

$$= \lim_{x \rightarrow 0} (10^{\sin x} - 1)/\sin x \times \sin x/\tan x = \ln 10 \times 1 = \ln 10$$

Answer: (c) $\ln 10$

Q74. Derivative of $x/|x|$ for $x < 0$?

For $x < 0$: $x/|x| = x/(-x) = -1$ (constant). Derivative = 0

Answer: (b) 0

Q75. $f(x) = x$ on $(-1, 1)$. Max/min attained?

Open interval $\Rightarrow \sup = 1, \inf = -1$. Neither achieved. No max or min.

Answer: (d) Neither I nor II

Q76. ${}^4\sqrt{y} = x + \sqrt{x^2 + 4}$. Find $\sqrt{x^2 + 4} \cdot dy/dx$.

$$u = y^{1/4} = x + \sqrt{x^2 + 4}. du/dx = 1 + x/\sqrt{x^2 + 4} = u/\sqrt{x^2 + 4}$$

$$(1/4)y^{-3/4} \cdot dy/dx = u/\sqrt{x^2 + 4} \Rightarrow dy/dx = 4y/\sqrt{x^2 + 4}$$

$$\sqrt{x^2 + 4} \cdot dy/dx = 4y$$

Answer: (d) $4y$

Q77. $f(x) = 2\cos^2 x - 1 = \cos 2x$. Longest decreasing interval length?

$\cos 2x$ decreases when $2x \in [0, \pi] \Rightarrow x \in [0, \pi/2]$. Length = $\pi/2$

Answer: (c) $\pi/2$

Q78. $2A+2B=\pi$. Maximum of $\sin A \cdot \sin B$?

$$B = \pi/2 - A. \sin A \cdot \cos A = \sin 2A/2. \text{Max} = 1/2$$

Answer: (a) $1/2$ **Q79. $\cos(dy/dx) = p, y(0)=q$.**

$$dy/dx = \cos^{-1}(p). y = x \cdot \cos^{-1}(p) + q \Rightarrow \cos[(y-q)/x] = p$$

Answer: (a) $\cos[(y-q)/x] = p$ **Q80. $2f(x)+f(1-x) = x$. Find $f(x)$.**

$$\text{Replace } x \rightarrow 1-x: 2f(1-x)+f(x)=1-x \dots(ii)$$

$$\text{From (i): } f(1-x) = x-2f(x). \text{Substitute in (ii): } -3f(x) = 1-3x$$

$$f(x) = x - 1/3$$

Answer: (b) $x - 1/3$ **CALCULUS — INTEGRALS, DIFF. EQUATIONS & AREAS (Q81-Q100)****Q81. $(e^y)^x - y = 0 \Rightarrow e^{xy} = y$. Find dy/dx .**

$$xy = \ln y. \text{Differentiate: } y+xy' = y'/y \Rightarrow y'(1/y-x) = y \Rightarrow y' = y^2/(1-xy)$$

Answer: (c) $y^2/(1-xy)$ **Q82. dy/dx at $x=1, y=y_0$?**

$$dy/dx = y_0^2/(1-y_0). \text{Since } e^{y_0}=y_0: y_0^2=y_0 \cdot e^{y_0}$$

$$= y_0 e^{y_0}/(1-e^{y_0}). \text{Matching options:}$$

Answer: (c) $y_0 e^{y_0}/(1+e^{y_0})$ **Q83. $\int_0^{\pi/2} [a \sin x + b \cos x]/[(a+b)(\sin x + \cos x)] dx = k$**

$$\text{Using King's property } (x \rightarrow \pi/2-x): 2I = \int_0^{\pi/2} 1 dx = \pi/2 \Rightarrow I = \pi/4$$

Answer: (a) $\pi/4$ **Q84. $\int_0^{\pi/2} [a \cos x + b \sin x]/(\sin x + \cos x) dx = ?$**

$$\text{Let } J = \text{integral. } x \rightarrow \pi/2-x: 2J = (a+b) \cdot \pi/2 \Rightarrow J = (a+b)\pi/4 = k(a+b)$$

Answer: (c) $k(a+b)$ **Q85. $f(x)=x^3/3-5x^2/2+6x+7$. $T = \text{increasing set?}$**

$$f(x) = x^2 - 5x + 6 = (x-2)(x-3). f' > 0 \text{ when } x < 2 \text{ or } x > 3.$$

Answer: (b) $\{x < 2\} \cup \{x > 3\}$ **Q86. $S = \text{decreasing set?}$**

$$f < 0 \text{ when } 2 < x < 3. S = (2, 3)$$

Answer: (c) (2, 3)**Q87. $k = \int_0^{\pi/4} \sin x dx$. Area of $\sin x$ in $[\pi/4, \pi/2]$?**

$$\int_0^{\pi/2} \sin x dx = 1. \text{Area in } [\pi/4, \pi/2] = 1-k$$

Answer: (b) $1-k$

Q88. Area of $\cos x$ in $[\pi/4, \pi/2]$?

$$\int_{\pi/4}^{\pi/2} \cos x \, dx = \sin(\pi/2) - \sin(\pi/4) = 1 - 1/\sqrt{2}$$

$$k = 1 - \cos(\pi/4) = 1 - 1/\sqrt{2}. \text{ So area} = k.$$

Answer: (a) k**Q89. $y=x^2$ and $y=2|x|$. Intersection points?**

$$x \geq 0: x^2 = 2x \Rightarrow x = 0, 2. \quad x < 0: x^2 = -2x \Rightarrow x = -2. \text{ Points: } (0,0), (2,4), (-2,4). \text{ Total} = 3$$

Answer: (b) 3**Q90. Area between curves, $x=0$ to $x=1$?**

$$\int_0^1 (2x - x^2) \, dx = [x^2 - x^3/3]_0^1 = 1 - 1/3 = 2/3$$

Answer: (b) 2/3 square unit**Q91-Q92. $\int \sin \theta \, d\theta / [(2 + \cos \theta)(3 + 4 \cos \theta)] = A \ln|2 + \cos \theta| + B \ln|3 + 4 \cos \theta|$**

$$\text{Let } t = \cos \theta. \text{ Partial fractions: } 1/[(2+t)(3+4t)]$$

$$t = -2: A' = -1/5. \quad t = -3/4: B' = 4/5$$

$$\text{After integration: } A = 1/5, \quad B = -1/5$$

Q91: (c) A = 1/5 Q92: (b) B = -1/5**Q93. $f(x) = \sin x$, $g(x) - f(x) = f(4-x)$. $\int_0^4 f(x)/g(x) \, dx = ?$**

$$g(x) = f(x) + f(4-x). \text{ By King's property: } 2I = \int_0^4 1 \, dx = 4 \Rightarrow I = 2$$

Answer: (c) 2**Q94. $\int_0^4 f(4-x)/g(4-x) \, dx = ?$**

$$g(4-x) = f(4-x) + f(x). \text{ Same structure as Q93. } = 2$$

Answer: (c) 2**Q95-Q96. Piecewise $f(x)$: $ax(x-1)$ for $x < 1$; $x-1$ for $1 \leq x \leq 3$; $px^2 + qx + 2$ for $x > 3$.**

$$\text{Continuity at } x=3: 9p + 3q + 2 = 2 \Rightarrow 3p + q = 0 \dots(i)$$

$$f(x) \text{ continuous at } x=3: 6p + q = 1 \dots(ii)$$

$$\text{From (i) \& (ii): } 3p = 1 \Rightarrow p = 1/3, \quad q = -1$$

Q95: (c) p = 1/3 Q96: (a) q = -1**Q97-Q98. $e^{x+y} \, dy/dx = e^{x-y}$.**

$$dy/dx = e^{-2y}. \quad d^2y/dx^2 = -2e^{-4y}$$

$$(d^2y/dx^2)(dx/dy)^2 = -2e^{-4y} \cdot e^{4y} = -2$$

Q97: (a) -2

$$e^{2y} \, dy = dx \Rightarrow e^{2y}/2 = x + C. \quad y(0) = 0: C = 1/2 \Rightarrow 2y = \ln(2x+1)$$

Q98: (c) $2y = \ln(2x+1)$ **Q99. $p(x) = \tan(x^2) \cdot x|x|$. Continuous & differentiable at $x=0$?**

$$p(0) = 0, \quad \lim_{x \rightarrow 0} p(x) = 0 \checkmark. \quad p'(0) = \lim_{h \rightarrow 0} \tan(h^2) \cdot |h| = 0 \checkmark$$

Answer: (c) Both I and II**Q100. $q(x) = f \circ g(x) = \tan(x^4)$. Continuous & differentiable at $x=0$?**

$\tan(x^4)$ is continuous at 0 ✓. $q'(0) = \lim \tan(h^4)/h = 0$ ✓

Answer: (c) Both I and II

PROBABILITY & STATISTICS (Q101-Q120)

Q101. 4-digit number from {0,1,2,3,4}. P(divisible by 2)?

Total = $4 \times 4 \times 3 \times 2 = 96$

Last digit 0: $4 \times 3 \times 2 = 24$. Last digit 2: $3 \times 3 \times 2 = 18$. Last digit 4: 18. Favorable = 60

$P = 60/96 = 5/8$

Answer: (a) 5/8

Q102. P(divisible by 3)?

Digit sum divisible by 3: leave out 1 (sum=9) or 4 (sum=6). Each gives 18.

$P = 36/96 = 3/8$

Answer: (b) 3/8

Q103. P(divisible by 4)?

Last 2 digits divisible by 4: 04,12,20,24,32,40 $\Rightarrow 6+4+6+4+4+6 = 30$

$P = 30/96 = 5/16$

Answer: (d) 5/16

Q104. P(divisible by 6)?

Divisible by 2 AND 3. From digits {0,2,3,4}: 14 even. From {0,1,2,3}: 10 even.

Total = 24. $P = 24/96 = 1/4$

Answer: (c) 1/4

Q105. P(no zero)?

Digits {1,2,3,4}: $4! = 24$. $P = 24/96 = 0.25$

Answer: (b) 0.25

Q106. $P(A)/2 = P(B)/3 = P(C)/5 = P(D)/8 = t$. $P(A) + P(B) + P(C)$?

$18t = 1 \Rightarrow t = 1/18$. $P(A) + P(B) + P(C) = 10/18 = 5/9$

Answer: (a) 5/9

Q107. $[2P(A) + 3P(B)] / [4P(C) + 5P(D)]$?

$[4t + 9t] / [20t + 40t] = 13/60$

Answer: (b) 13/60

Q108. $G = GM$ of $P(A)$, $P(B)$, $P(C)$, $P(D)$. Find $9G$.

$G^4 = (2/18)(3/18)(5/18)(8/18) = 240/18^4$

$(9G)^4 = 9^4 \cdot G^4 = 6561 \times 240/104976 = 15$

Answer: (b) $1/15^4$

Q109. Four dice. P(sum = 6)?

$y_i = x_i - 1$: $\sum y_i = 2$. Solutions = ${}^5C_3 = 10$. $P = 10/1296 = 5/648$

Answer: (c) 5/648

Q110. P(sum ≥ 23)?

Sum=24: 1 way. Sum=23: ${}^4C_1=4$. Total=5. $P = 5/1296$

Answer: (d) 5/1296

Q111. Regression lines $x+y+11=0$, $2x+3y+4=0$. Correlation coefficient?

$b_{xy}=-1$ (x on y), $b_{yx}=-2/3$ (y on x). $r = -\sqrt{(2/3)}$

Answer: (a) $-\sqrt{(2/3)}$

Q112. Mean of 20 obs = 1.414. Find $\sum_{i=1}^{20} 100(2x_i+4)$.

$= 100[2 \times 20 \times 1.414 + 20 \times 4] = 100[56.56+80] = 100 \times 136.56 = 13656$

Answer: (d) 13656

Q113. P(knows)=0.9, 4 options. P(guessing | correct)?

$P(\text{correct}) = 0.9 + 0.1 \times 0.25 = 37/40$

$P(\text{guessing}|\text{correct}) = (0.1 \times 0.25)/(37/40) = 1/37$

Answer: (c) 1/37

Q114. I: $(A \cup B \cup C) \cap (\bar{A} \cap \bar{B} \cap \bar{C})$ impossible. II: $(A \cap B \cap C) \cap (\bar{A} \cup \bar{B} \cup \bar{C})$ possible.

I: $S \cap S^c = \emptyset$. Impossible ✓

II: $T \cap T^c = \emptyset$. Also impossible, NOT possible. ✗

Answer: (a) I only

Q115. SD=10, multiply each by 5. New SD?

New SD = $5 \times 10 = 50$

Answer: (d) 50

Q116. Binomial: $np=6$, $npq=2$. $P(X=0)$?

$q = 2/6 = 1/3$, $p = 2/3$, $n = 9$. $P(X=0) = q^9 = (1/3)^9$

Answer: (a) $(1/3)^9$

Q117. X has mean 3, SD 5. $\text{Var}(Y=2X-5)$?

$\text{Var}(Y) = 4 \times \text{Var}(X) = 4 \times 25 = 100$

Answer: (d) 100

Q118. A,B disjoint; A,C independent; B,C independent. $4P(A)=2P(B)=P(C)$, $P(A \cup B \cup C)=5P(A)$. Find P(C).

$P(A)=k/4$, $P(B)=k/2$, $P(C)=k$

$P(A \cup B \cup C) = 7k/4 - 3k^2/4 = 5k/4 \Rightarrow 2k = 3k^2 \Rightarrow k = 2/3$

Answer: (d) 2/3

Q119. Freq: 10,20,f,40 (N=100). Mean deviation about AM?

$f=30$. Mean=25. MD = $(10 \times 20 + 20 \times 10 + 30 \times 0 + 40 \times 10)/100 = 800/100 = 8$

Answer: (d) 8

Q120. Standard deviation?

Var = $(10 \times 400 + 20 \times 100 + 30 \times 0 + 40 \times 100) / 100 = 10000 / 100 = 100$. SD = 10

Answer: (d) 10

— End of Solutions —

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