

NDA MATHEMATICS - SET B

COMPLETE SOLUTIONS | 120 Questions | Answer Key

CALCULUS, LIMITS & INTEGRATION (Q1 - Q20)

Q1. Value of A in partial fraction integral

Let $u = \cos \theta$. Partial fractions: $\frac{1}{(2+u)(3+4u)} = \frac{-1/5}{(2+u)} + \frac{4/5}{(3+4u)}$. After integrating with sign change from $du = -\sin \theta d\theta$: $A = 1/5$.

Answer : (c) 1/5

Q2. Value of B

From same decomposition: $B = -1/5$.

Answer : (b) -1/5

Q3. Integral of $f(x)/g(x)$ from 0 to 4, $f(x)=\sin x$, $g(x)=\sin x + \sin(4-x)$

King's property: $I = \int_0^4 \frac{\sin x}{\sin x + \sin(4-x)} dx$. Replace x with $4-x$: $I = \int_0^4 \frac{\sin(4-x)}{\sin(4-x) + \sin x} dx$. Adding: $2I = \int_0^4 1 dx = 4$. So $I = 2$.

Answer : (c) 2

Q4. Integral of $f(4-x)/g(4-x)$ from 0 to 4

$g(4-x) = \sin(4-x) + \sin x$ (same denominator). By King's property this integral also = 2.

Answer : (c) 2

Q5. Value of p (piecewise continuity / differentiability)

Continuity at $x=3$: $9p + 3q + 2 = 2$, so $3p + q = 0$. $f'(x)$ continuous at $x=3$: $6p + q = 1$. Solving: $3p = 1$, $p = 1/3$.

Answer : (c) 1/3

Q6. Value of q

From $3p + q = 0$ and $p = 1/3$: $q = -1$.

Answer : (a) -1

Q7. $(d^2 y / dx^2) * (dx/dy)^2$ for the given ODE

$dy/dx = e^{-2y}$. $d^2 y/dx^2 = -2 e^{-4y}$. $(dx/dy)^2 = e^{4y}$. Product = $-2 e^{-4y} * e^{4y} = -2$.

Answer : (a) -2

Q8. Solution of $e^{(2y)} dy/dx = 1$, $y(0)=0$

Separating: $e^{(2y)} dy = dx$. Integrating: $(1/2) e^{(2y)} = x + C$. $y(0)=0$ gives $C=1/2$. So $e^{(2y)} = 2x + 1$, i.e. $2y = \ln(2x+1)$.

Answer : (c) $2y = \ln(2x+1)$

Q9. $p(x) = \tan(x^2) * x|x|$: continuity and differentiability at $x = 0$

For $x > 0$: $p = x^2 \tan(x^2)$. For $x < 0$: $p = -x^2 \tan(x^2)$. $p(0) = 0$, limits from both sides = 0 (continuous). $p'(0+) = \lim h * \tan(h^2) = 0 = p'(0-)$ (differentiable).

Answer : (c) Both I and II

Q10. $q(x) = f(g(x)) = \tan(x^4)$ at $x = 0$

$q(x) = \tan(x^4)$ for all x . $q(0)=0$ (continuous). $q'(x) = 4x^3 \sec^2(x^4)$, $q'(0) = 0$ (differentiable).

Answer : (c) Both I and II

Q11. If $f(x)$ is differentiable at $x = a$...

I. f is continuous at $x = a$ (standard theorem - TRUE). II. $\lim_{x \rightarrow a} f(x) = f(a)$ (definition of continuity - TRUE).

Answer : (c) Both I and II

Q12. $\lim_{x \rightarrow 1} [x^{(n^2 - 1)} - 1] / [x^{(n+1)} - 1]$

Using $\lim_{x \rightarrow 1} (x^k - 1)/(x - 1) = k$: Ratio = $(n^2 - 1)/(n + 1) = (n-1)(n+1)/(n+1) = n - 1$.

Answer : (c) $n - 1$

Q13. $\lim_{x \rightarrow 0} (10^{\sin x} - 1) / \tan x$

= $\lim [(10^{\sin x} - 1)/\sin x] * [\sin x / \tan x] = \ln 10 * 1 = \ln 10$.

Answer : (c) $\ln 10$

Q14. Derivative of $x / |x|$ for $x < 0$

For $x < 0$: $x/|x| = x/(-x) = -1$ (constant). Derivative = 0.

Answer : (b) 0

Q15. $f(x) = x$ on open interval $(-1, 1)$: max / min ?

Open interval -- endpoints not included. f approaches 1 and -1 but never attains. Neither max nor min attained.

Answer : (d) Neither I nor II

Q16. 4th-root(y) = $x + \sqrt{x^2 + 4}$, find $\sqrt{x^2 + 4} * dy/dx$

$y = [x + \sqrt{x^2 + 4}]^4$. $dy/dx = 4y / \sqrt{x^2 + 4}$. Therefore $\sqrt{x^2 + 4} * dy/dx = 4y$.

Answer : (d) $4y$

Q17. Longest interval where $f(x) = 2 \cos^2 x - 1$ is decreasing

$f(x) = \cos 2x$. $f'(x) = -2 \sin 2x < 0$ when $\sin 2x > 0$, i.e. x in $(0, \pi/2)$. Length = $\pi/2$.

Answer : (c) $\pi/2$

Q18. $2A + 2B = \pi$, maximum of $\sin A * \sin B$

$A + B = \pi/2$. $\sin A * \sin B = \sin A * \cos A = (1/2) \sin 2A$. Max when $\sin 2A = 1$. Max value = $1/2$.

Answer : (a) $1/2$

Q19. $\cos(dy/dx) = p$, $y(0) = q$: solution

$dy/dx = \cos^{-1}(p)$. $y = x * \cos^{-1}(p) + q$. Rearranging: $\cos((y-q)/x) = p$.

Answer : (a) $\cos((y-q)/x) = p$

Q20. $2f(x) + f(1-x) = x$, find $f(x)$

Replace x by $(1-x)$: $2f(1-x) + f(x) = 1-x$. From both equations: $3f(x) = 3x - 1$. $f(x) = x - 1/3$.

Answer : (b) $x - 1/3$

COORDINATE GEOMETRY & 3D (Q21 - Q30)

Q21. Angle alpha between line through origin and line L

L : slope = $-1/\sqrt{3}$. Other line slope = $1/\sqrt{3}$. $\tan \alpha = |(-1/\sqrt{3} - 1/\sqrt{3})/(1 + (-1/3))| = |(-2/\sqrt{3})/(2/3)| = \sqrt{3}$. $\alpha = 60$ deg.

Answer : (c) 60 deg

Q22. Angle line L makes with positive y-axis

Angle with x-axis = 150 deg. Angle with y-axis = $|150 - 90| = 60$ deg.

Answer : (c) 60 deg

Q23. PQ parallel to RS? PR perpendicular to QS?

PQ = (6,2,0), RS = (-4,-2,2): ratios unequal -- NOT parallel. PR = (8,-1,3), QS = (-2,-5,5): dot product = 4 -- NOT perpendicular.

Answer : (d) Neither I nor II

Q24. $PQ^2 + 2QS^2 - 2PR^2$

$PQ^2 = 40$, $QS^2 = 54$, $PR^2 = 74$. $40 + 108 - 148 = 0$.

Answer : (b) 0

Q25. Line through (-1,2,-3) perpendicular to plane $2x+3y+z+5=0$

Direction = normal = (2,3,1). $(x+1)/2 = (y-2)/3 = (z+3)/1$. Multiply: $3(x+1) = 2(y-2) = 6(z+3)$.

Answer : (c) $3x+3 = 2y-4 = 6z+18$

Q26. Direction ratios of line M parallel to plane

Must be perpendicular to normal (2,3,1). Check (2,2,-10): $2*2 + 3*2 + 1*(-10) = 0$.

Answer : (d) $\langle 2, 2, -10 \rangle$

Q27. X(a,p), Y(b,q), Z(c,r) with a,b,c and p,q,r in AP

$2b = a+c$, $2q = p+r$. Y = midpoint of XZ. Points are collinear.

Answer : (a) on a straight line

Q28. If p,q,r not in AP and $b = c$

Y and Z have same x-coordinate. Line YZ is parallel to y-axis.

Answer : (a) y-axis

Q29. A(0,2), B(2,3), C(4,5), D(0,k) on circle : values of k

Circle: $x^2 + y^2 + 5x - 19y + 34 = 0$. D(0,k): $k^2 - 19k + 34 = 0$. $k = 2$ or $k = 17$.

Answer : (c) 2, 17

Q30. Diameter of circle through A, B, D

Centre $(-5/2, 19/2)$, $r^2 = 250/4$. Diameter = $2r = \sqrt{250} = 5\sqrt{10}$.

Answer : (b) $5\sqrt{10}$

PROBABILITY & STATISTICS (Q31 - Q50)

Q31. 4-digit number from {0,1,2,3,4} no repetition, divisible by 2

Total = 96. Last digit in {0,2,4}: $24+18+18 = 60$. $P = 60/96 = 5/8$.

Answer : (a) 5/8

Q32. Divisible by 3

Leave out digit with $d = 1 \pmod{3}$: $d=1$ or $d=4$. $18+18 = 36$. $P = 36/96 = 3/8$.

Answer : (b) 3/8

Q33. Divisible by 4

Valid endings {04,12,20,24,32,40}: $6+4+6+4+4+6 = 30$. $P = 30/96 = 5/16$.

Answer : (d) 5/16

Q34. Divisible by 6

Both by 2 and 3. From sets {0,2,3,4} and {0,1,2,3}: $14+10 = 24$. $P = 24/96 = 1/4$.

Answer : (c) 1/4

Q35. Number does not contain zero

Digits {1,2,3,4}: $4! = 24$. $P = 24/96 = 0.25$.

Answer : (b) 0.25

Q36. $P(A)+P(B)+P(C)$ where $P(A)/2 = P(B)/3 = P(C)/5 = P(D)/8 = k$

$18k = 1$, $k = 1/18$. $P(A)+P(B)+P(C) = (2+3+5)k = 10/18 = 5/9$.

Answer : (a) 5/9

Q37. $[2P(A)+3P(B)] / [4P(C)+5P(D)]$

$= (4k + 9k) / (20k + 40k) = 13/60$.

Answer : (b) 13/60

Q38. $9G$ where $G =$ geometric mean of $P(A), P(B), P(C), P(D)$

$G^4 = (2k)(3k)(5k)(8k) = 240 k^4$. $9G = (240)^{1/4} / 2 = 2*(15)^{1/4} / 2 = 15^{1/4}$.

Answer : (b) $15^{1/4}$

Q39. $P(\text{sum} = 6)$ with four dice

Each die ≥ 1 . Let $e_j = d_j - 1 \geq 0$: $e_1+e_2+e_3+e_4 = 2$. $C(5,2) = 10$. $P = 10/1296 = 5/648$.

Answer : (c) 5/648

Q40. $P(\text{sum} \geq 23)$ with four dice

By symmetry $P(\text{sum} \geq 23) = P(\text{sum} \leq 5)$. $P(4)=1$, $P(5)=4$. Total = 5/1296.

Answer : (d) 5/1296

Q41. Correlation coefficient from regression lines

$b_{yx} = -2/3$, $b_{xy} = -1$. $r^2 = (-2/3)(-1) = 2/3$. $r = -\sqrt{2/3}$ (both negative).

Answer : (a) $-\sqrt{2/3}$

Q42. Sum of $100(2x_j + 4)$ where mean of 20 obs = 1.414

Sum $x_j = 28.28$. Total = $100 * [2(28.28) + 80] = 100 * 136.56 = 13656$.

Answer : (d) 13656

Q43. Bayes' theorem : P(guessing | correct)

$P(\text{knows}) = 0.9$, $P(\text{guess}) = 0.1$, $P(\text{correct}|\text{guess}) = 1/4$. $P = (0.1 * 0.25) / (0.9 + 0.025) = 1/37$.

Answer : (c) 1/37

Q44. (A U B U C) intersection (A' inter B' inter C') impossible?

I. $S \text{ inter } S' = \text{empty}$ -- impossible. TRUE. II. $T \text{ inter } T' = \text{empty}$ -- impossible, NOT possible. So II is wrong.

Answer : (a) I only

Q45. SD = 10, each observation multiplied by 5

New SD = $5 * 10 = 50$.

Answer : (d) 50

Q46. Binomial: mean=6, SD=sqrt(2). P(X=0)?

$np = 6$, $npq = 2$. $q = 1/3$, $p = 2/3$, $n = 9$. $P(X=0) = (1/3)^9$.

Answer : (a) (1/3)^9

Q47. Var(Y) where Y = 2X - 5, SD(X) = 5

$\text{Var}(Y) = 4 * \text{Var}(X) = 4 * 25 = 100$.

Answer : (d) 100

Q48. P(C) given conditions on A, B, C

$P(A) = x$, $P(B) = 2x$, $P(C) = 4x$. $7x - 12x^2 = 5x$. $x = 1/6$. $P(C) = 4/6 = 2/3$.

Answer : (d) 2/3

Q49. Mean deviation about AM (frequency distribution)

$f = 30$, Mean = 25. MD = $(200 + 200 + 0 + 400)/100 = 8$.

Answer : (d) 8

Q50. Standard deviation

$\text{Var} = (4000 + 2000 + 0 + 4000)/100 = 100$. SD = 10.

Answer : (d) 10

MATRICES & DETERMINANTS (Q51 - Q70)

Q51. $|M + M^T|$ properties for 2×2 matrix

I. For symmetric: $|2M| = 4|M| \neq 2|M|$. FALSE. II. For anti-symmetric: $M + M^T = 0$, $|0| = 0$. TRUE.

Answer : (b) II only

Q52. $M^3 = M$, how many values of $|M|$?

$|M|^3 = |M|$. $|M|(|M|^2 - 1) = 0$. $|M|$ in $\{0, 1, -1\}$. Three values.

Answer : (c) Three

Q53. Determinant $|x \ 1 \ 1 ; 1 \ y \ 1 ; 1 \ 1 \ z| > 0$

Expanding: $xyz - (x+y+z) + 2 = q - p + 2 > 0$. So $q + 2 > p$.

Answer : (c) $q + 2 > p$

Q54. Set operation simplification

$(A \cap B) \cup (A - B) = A$. $(A \cap B) \cup (B - A) = B$. Result = $(A - B) \cup A = A$.

Answer : (b) A

Q55. $A \cap C = B \cap C = \text{empty}$, $A \cup C = B \cup C$

Any x in A : x in $A \cup C = B \cup C$, x not in $C \Rightarrow x$ in B . So $A \subset B$. Similarly $B \subset A$. $A = B$. C need not be empty.

Answer : (b) II only

Q56. $2 \sec 4\beta = \tan 2\alpha + \cot 2\alpha$

RHS = $2 / \sin 4\alpha = 2 \csc 4\alpha$. $\sec 4\beta = \csc 4\alpha$. $4(\alpha + \beta) = \pi/2$. $\alpha + \beta = \pi/8$.

Answer : (d) $\pi/8$

Q57. Complementary angles: $\alpha - \beta = \pi/6$, $m \tan \beta = n \tan \alpha$

$\alpha = \pi/3$, $\beta = \pi/6$. $n/m = \tan(\pi/6)/\tan(\pi/3) = 1/3$. $(m+n)/(m-n) = (4/3)/(2/3) = 2$.

Answer : (a) 2

Q58. $x = \sec \theta - \tan \theta$, $y = \operatorname{cosec} \theta + \cot \theta$

Using $t = \tan(\theta/2)$: $x = (1-t)/(1+t)$, $y = 1/t$. Eliminating t : $x + y - xy - 1 = 0$.

Answer : (a) $x + y - xy - 1 = 0$

Q59. $\cos \theta = 1/3$, find $\sin(\theta/2) \sin(3\theta/2)$

$= (1/2)[\cos \theta - \cos 2\theta] = (1/2)[1/3 - (2/9 - 1)] = (1/2)(10/9) = 5/9$.

Answer : (a) 5/9

Q60. Maximum of $\cos x + \sqrt{3} \sin x$

$= 2 \cos(x - \pi/3)$. Maximum when $x = \pi/3$.

Answer : (b) $\pi/3$

Q61. Quadratics unchanged by squaring roots (leading coeff = 1)

Roots from $\{0, 1, \omega, \omega^2\}$. Quadratics: x^2 , $x^2 - x$, $(x-1)^2$, $x^2 + x + 1$. Total = 4.

Answer : (b) 4

Q62. Roots of $x^2 - (b+c)x + bc = 0$

AM of original roots = $b = A$, GM = $c = G$. New equation = $(x-b)(x-c) = 0$. Roots are A, G.

Answer : (a) A, G

Q63. $1 - \log_{10}(2) = \log_{10}(5^x + 4^x + 3^x + 2^x + 1)$

LHS = $\log_{10}(5)$. At $x=0$: RHS = $\log_{10}(5)$. So $x = 0$.

Answer : (d) 0

Q64. $f(x) = \det[3x^2, \cos x, \sin x; 6, -1, 0; q, q^2, q^3]$. Find $f''(0)$

Expanding and differentiating twice: $f''(0) = -6q^3 + 6q^3 = 0$.

Answer : (b) 0

Q65. Determinant with row differences : value of k

$R_1 + R_2 + R_3 = (0,0,0)$ always. LHS det = 0 always. $k = 0$.

Answer : (b) 0

Q66. Determinant with cube roots of unity

$p^2 + q^2 + r^2 = 0$. After simplification: $\det = 4(pqr)^2 = 4(1)^2 = 4$.

Answer : (d) 4

Q67. $|A A^T|$ where $|A| = -2$

$|A A^T| = |A| * |A^T| = |A|^2 = (-2)^2 = 4$.

Answer : (d) 4

Q68. Symmetric inverse and singular adjoint

I. Inverse of symmetric is symmetric. TRUE. II. $|\text{adj } A| = |A|^{(n-1)} = 0$ for singular A. TRUE.

Answer : (c) Both I and II

Q69. $M = 2 * I_3 : |M| * |\text{adj } M|$

$|M| = 8$. $|\text{adj } M| = 8^2 = 64$. Product = $8 * 64 = 512$.

Answer : (d) 512

Q70. $M_k = [[k, k-1], [k-1, k]]$, Sum $|M_k|$ from $k=1$ to 50

$|M_k| = k^2 - (k-1)^2 = 2k - 1$. Sum = $1+3+5+\dots+99 = 50^2 = 2500$.

Answer : (c) 2500

SEQUENCES, COMPLEX NUMBERS & COMBINATORICS (Q71 - Q84)

Q71. AP/GP ratio problem

$d = p, q = 2p, r = 3p. (p+q):(q+r):(r+p) = 3p : 5p : 4p = 3:5:4.$

Answer : (c) 3 : 5 : 4

Q72. GP means : $g_1^2/g_2 + g_2^2/g_1$

$g_1^2/g_2 = p, g_2^2/g_1 = q. \text{Sum} = p + q = 2m.$

Answer : (b) 2m

Q73. Complex inequalities validity

Complex numbers cannot be ordered. None valid.

Answer : (a) None

Q74. $|Z_1 + Z_2| / |Z_1 - Z_2|$ where $Z_1/(4Z_2)$ purely imaginary

$Z_1/Z_2 = ki. |1+ki| / |1-ki| = \sqrt{1+k^2} / \sqrt{1+k^2} = 1.$

Answer : (d) 1

Q75. Cube roots of -8 : ratio

$\alpha = -2, \beta = -2w, \gamma = -2w^2. \text{Numerator} = w * \text{Denominator}. \text{Ratio} = w = \gamma/\beta.$

Answer : (b) γ / β

Q76. $S_n = 3n^2 + 5n$, find m where $a_m = 68$

$a_m = S_m - S_{(m-1)} = 6m + 2. 6m + 2 = 68. m = 11.$

Answer : (c) 11

Q77. Set S with $(2n+1)$ elements, subsets with at most n elements = 1024

By symmetry: $2^{(2n)} = 1024 = 2^{10}. 2n = 10. n = 5.$

Answer : (d) 5

Q78. Max intersection points of 5 circles

Each pair intersects in at most 2 points. $C(5,2) * 2 = 10 * 2 = 20.$

Answer : (c) 20

Q79. Greatest r satisfying ${}^{15}C_{(r+1)} > 2 * {}^{15}C_r$

$(15-r)/(r+1) > 2. r < 13/3 = 4.33. \text{Greatest } r = 4.$

Answer : (c) 4

Q80. $n = m \cdot C_2$, find $n \cdot C_2$

$n(n-1)/2 = m(m-1)(m+1)(m-2)/8 = 3 * (m+1) \cdot C_4.$

Answer : (c) $3 * (m+1) \cdot C_4$

Q81. Ellipse and hyperbola with coinciding foci : value of p

Ellipse foci = $\sqrt{16-p}$. Hyperbola foci = 3. $16 - p = 9. p = 7.$

Answer : (c) 7

Q82. Difference of eccentricities

$e(\text{hyp}) = 5/4$, $e(\text{ell}) = 3/4$. Difference = $1/2 = 0.5$.

Answer : (a) 0.5

Q83. Radius of concentric sphere through origin

Centre (2,3,6). Distance to origin = $\sqrt{4+9+36} = 7$.

Answer : (c) 7

Q84. Radius = 8, find k

$49 - k = 64$. $k = -15$.

Answer : (a) -15

VECTORS & MORE CALCULUS (Q85 - Q100)

Q85. Angle between a and b (from $\mathbf{a} \times \mathbf{b} = \mathbf{c}$, $\mathbf{b} \times \mathbf{c} = \mathbf{a}$)

Cross product relations give a perp b. Angle = $\pi/2$.

Answer : (c) $\pi/2$

Q86. Angle between a and c

$\mathbf{a} \times \mathbf{b} = \mathbf{c}$ implies c perp a. Angle = $\pi/2$.

Answer : (c) $\pi/2$

Q87. Are a, b, c orthogonal ? Unit vectors ?

I. All mutually orthogonal (TRUE). II. From magnitudes: $|\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|=1$ (TRUE).

Answer : (c) Both I and II

Q88. Vector identity verification

$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = |\mathbf{c}|^2 = 1$. $(\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a} = |\mathbf{a}|^2 = 1$. Both statements hold.

Answer : (c) Both I and II

Q89. $\mathbf{c} = p\mathbf{a} + q\mathbf{b} + r(\mathbf{a} \times \mathbf{b})$: find p + q

$p = \mathbf{a} \cdot \mathbf{c} = \cos(\pi/3) = 1/2$. $q = \mathbf{b} \cdot \mathbf{c} = \cos(\pi/3) = 1/2$. $p + q = 1$.

Answer : (b) 1

Q90. Value of r^2

$|\mathbf{c}|^2 = p^2 + q^2 + r^2 = 1$. $1/4 + 1/4 + r^2 = 1$. $r^2 = 1/2$.

Answer : (d) 1/2

Q91. $\frac{dy}{dx}$ for $(e^y)^x - y = 0$

$e^{(xy)} = y$. $xy = \ln y$. Differentiating: $y + xy' = y'/y$. $\frac{dy}{dx} = \frac{y^2}{1 - xy}$.

Answer : (c) $\frac{y^2}{1 - xy}$

Q92. $\frac{dy}{dx}$ at $x = 1$, $y = y_0$

At $x=1$: $e^{(y_0)} = y_0$. $\frac{dy}{dx} = \frac{y_0^2}{1-y_0} = \frac{y_0 * e^{(y_0)}}{1 - e^{(y_0)}}$.

Answer : (d) $\frac{y_0 * e^{(y_0)}}{1 - e^{(y_0)}}$

Q93. Value of k (integral with (a+b) denominator)

Using King's property: $I = (a+b) * \pi/4 / (a+b) = \pi/4$. $k = \pi/4$.

Answer : (a) $\pi/4$

Q94. Integral of $(a \cos x + b \sin x)/(\sin x + \cos x)$ from 0 to $\pi/2$

By King's property: $= (a+b) * \pi/4 = k * (a+b)$.

Answer : (c) $k(a+b)$

Q95. T : where $f(x) = x^3/3 - 5x^2/2 + 6x + 7$ increases

$f'(x) = (x-2)(x-3) \geq 0$ when $x \leq 2$ or $x \geq 3$.

Answer : (a) $\{x \leq 2\} \cup \{x \geq 3\}$

Q96. S : where f decreases

$f'(x) \leq 0$ when $2 \leq x \leq 3$.

Answer : (d) [2, 3]

Q97. Area : $\sin x$ from $\pi/4$ to $\pi/2$ (in terms of k)

$k = 1 - \sqrt{2}/2$. Integral $= \sqrt{2}/2 = 1 - k$.

Answer : (b) $1 - k$

Q98. Area : $\cos x$ from $\pi/4$ to $\pi/2$

Integral $= 1 - \sqrt{2}/2 = k$.

Answer : (a) k

Q99. Intersection points of $y = x^2$ and $y = 2|x|$

$x \geq 0$: $x=0, 2$. $x < 0$: $x=0, -2$. Distinct: $(0,0), (2,4), (-2,4)$. Total = 3.

Answer : (b) 3

Q100. Area bounded between $x=0$ and $x=1$

Integral(0 to 1) $(2x - x^2) dx = [x^2 - x^3/3] = 1 - 1/3 = 2/3$ sq unit.

Answer : (b) $2/3$ square unit

RELATIONS, ALGEBRA & TRIGONOMETRY (Q101 - Q120)

Q101. $R = \{(x,y) : x = y^3, x,y \in \mathbb{N}\}$: symmetric ? transitive ?

Not symmetric: (8,2) in R but (2,8) not in R. Not transitive: chain fails generally.

Answer : (d) Neither I nor II

Q102. Minimum of $x^2 + kx + k^2$

$f'(x) = 2x + k = 0$ at $x = -k/2$. $f(-k/2) = k^2/4 - k^2/2 + k^2 = 3k^2/4$.

Answer : (c) $3k^2/4$

Q103. $\sqrt{x} + x + 1 = 0$ and $5\sqrt{x} - x - 4 = 0$

I: $u^2 + u + 1 = 0$, $D < 0$, no real roots. FALSE. II: $(u-1)(u-4) = 0$, $x=1,16$. Two rational roots. TRUE.

Answer : (b) II only

Q104. Numbers > 1000 from $\{0,1,2,3\}$, no repetition

First digit in $\{1,2,3\}$: 3 choices. Remaining $3! = 6$. Total = 18.

Answer : (b) 18

Q105. AP : $T_p = k$, sum of $T_{(p+q)}$ and $T_{(p-q)}$

$a_{(p+q)} + a_{(p-q)} = 2[a + (p-1)d] = 2T_p = 2k$.

Answer : (a) $2k$

Q106. $(\sqrt{2}+1)^{10} = u+f$, $(\sqrt{2}-1)^{10} = v$: statements

$u+f+v = \text{integer (binomial)}$. $0 < f,v < 1$ so $f+v = 1$. Both I and II true.

Answer : (c) Both I and II

Q107. Multiplicative inverse of $(\sqrt{2}+1)^{20}$

$(\sqrt{2}+1)(\sqrt{2}-1) = 1$. Inverse = $(\sqrt{2}-1)^{20} = v^2$.

Answer : (c) v^2

Q108. Value of $v + f$

$f + v = 1$ (since $u+f+v$ is integer and $0 < f+v < 2$).

Answer : (b) 1

Q109. Value of u

$u + f + v = 2[1+90+840+1680+720+32] = 6726$. $u = 6726 - 1 = 6725$.

Answer : (d) 6725

Q110. Value of $u * v$

$uv = u/(u+f)$. Since $f > 0$, $uv < 1$. So $0 < uv < 1$.

Answer : (c) $0 < uv < 1$

Q111. theta in Q4, $3 \cot \theta + 4 = 0$: $\sin 2\theta + \cos 2\theta$

$\tan \theta = -3/4$. $\sin \theta = -3/5$, $\cos \theta = 4/5$. $\sin 2\theta = -24/25$, $\cos 2\theta = 7/25$. Sum = $-17/25$.

Answer : (b) $-17/25$

Q112. $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$

$\alpha - \beta = \pi$. $\cos 2\alpha + \cos 2\beta = -2 \cos(\alpha + \beta)$. Total = 0.

Answer : (a) 0

Q113. $\sin A = \cos B + \cos C$ in triangle : $\tan(B/2) + \cot(B/2)$

Condition gives $B = \pi/2$. $\tan(\pi/4) + \cot(\pi/4) = 1 + 1 = 2$.

Answer : (d) 2

Q114. Correct inverse trig identities

I. $\sin^{(-1)}(-x) = -\sin^{(-1)}(x)$ TRUE. IV. $\cot^{(-1)}(-x) = \pi - \cot^{(-1)}(x)$ TRUE. Two correct.

Answer : (b) Two

Q115. $\tan[2 \tan^{(-1)}(1/2) - \pi/4]$

$\tan(2\alpha) = 4/3$. $\tan(2\alpha - \pi/4) = (4/3 - 1)/(1 + 4/3) = (1/3)/(7/3) = 1/7$.

Answer : (d) 1/7

Q116. Triangle angles 1:1:4, longest side = 3 : perimeter

$A = B = \pi/6$, $C = 2\pi/3$. $a = 3 \sin(\pi/6)/\sin(2\pi/3) = \sqrt{3}$. Perimeter = $3 + 2\sqrt{3}$.

Answer : (b) $3 + 2\sqrt{3}$

Q117. $\sin(A-B) / \sin(A+B)$ in triangle

Using sine rule: $a^2 - b^2 = 4R^2 \sin(A-B) \sin(A+B)$. Ratio = $(a^2 - b^2) / c^2$.

Answer : (c) $(a^2 - b^2) / c^2$

Q118. Smallest x : $\log_{\sin x} \cos x + \log_{\cos x} \sin x = 2$

$t + 1/t = 2$. $t = 1$. $\sin x = \cos x$. $x = \pi/4$.

Answer : (c) $\pi/4$

Q119. Height of plane at 10 km, elevation 67.5 deg

$h = 10 \sin(67.5) = 10 \cos(22.5) = 10 \sqrt{(2 + \sqrt{2})/4} = 5 \sqrt{2 + \sqrt{2}}$ km.

Answer : (c) $5 \sqrt{2 + \sqrt{2}}$ km

Q120. Chord of unit circle subtending 45 deg at centre

Chord = $2 \sin(22.5) = 2 \sqrt{(2 - \sqrt{2})/4} = \sqrt{2 - \sqrt{2}}$ units.

Answer : (d) $\sqrt{2 - \sqrt{2}}$ units

COMPLETE ANSWER KEY -- NDA MATH SET B

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	(c)	21	(c)	41	(a)	61	(b)	81	(c)	101	(d)
2	(b)	22	(c)	42	(d)	62	(a)	82	(a)	102	(c)
3	(c)	23	(d)	43	(c)	63	(d)	83	(c)	103	(b)
4	(c)	24	(b)	44	(a)	64	(b)	84	(a)	104	(b)
5	(c)	25	(c)	45	(d)	65	(b)	85	(c)	105	(a)
6	(a)	26	(d)	46	(a)	66	(d)	86	(c)	106	(c)
7	(a)	27	(a)	47	(d)	67	(d)	87	(c)	107	(c)
8	(c)	28	(a)	48	(d)	68	(c)	88	(c)	108	(b)
9	(c)	29	(c)	49	(d)	69	(d)	89	(b)	109	(d)
10	(c)	30	(b)	50	(d)	70	(c)	90	(d)	110	(c)
11	(c)	31	(a)	51	(b)	71	(c)	91	(c)	111	(b)
12	(c)	32	(b)	52	(c)	72	(b)	92	(d)	112	(a)
13	(c)	33	(d)	53	(c)	73	(a)	93	(a)	113	(d)
14	(b)	34	(c)	54	(b)	74	(d)	94	(c)	114	(b)
15	(d)	35	(b)	55	(b)	75	(b)	95	(a)	115	(d)
16	(d)	36	(a)	56	(d)	76	(c)	96	(d)	116	(b)
17	(c)	37	(b)	57	(a)	77	(d)	97	(b)	117	(c)
18	(a)	38	(b)	58	(a)	78	(c)	98	(a)	118	(c)
19	(a)	39	(c)	59	(a)	79	(c)	99	(b)	119	(c)
20	(b)	40	(d)	60	(b)	80	(c)	100	(b)	120	(d)

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