

# NDA MATHEMATICS – SET D

COMPLETE SOLUTIONS • 120 Questions • Answer Key

## MATRICES, SETS & TRIGONOMETRY (Q1 - Q12)

### Q1. $|M + M^T|$ properties for $2 \times 2$ matrix

I. Symmetric:  $|2M| = 4|M| \neq 2|M|$ . FALSE. II. Anti-symmetric:  $M + M^T = O \Rightarrow |O| = 0$ . TRUE.

✓ Answer : (b) II only

### Q2. $M^3 = M$ : how many values of $|M|$ ?

$|M|^3 = |M| \Rightarrow |M|(|M|^2 - 1) = 0 \Rightarrow |M| \in \{0, 1, -1\}$ . Three values.

✓ Answer : (c) Three

### Q3. Determinant $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix} > 0$

Expanding:  $xyz - (x+y+z) + 2 = q - p + 2 > 0$ . So  $q + 2 > p$ .

✓ Answer : (c)  $q + 2 > p$

### Q4. $((A \cap B) \cup (A - B)) - ((A \cap B) \cup (B - A)) \cup A = ?$

$(A \cap B) \cup (A - B) = A$ .  $(A \cap B) \cup (B - A) = B$ .  $(A - B) \cup A = A$ .

✓ Answer : (b) A

### Q5. $A \cap C = B \cap C = \emptyset$ , $A \cup C = B \cup C$

$\forall x \in A : x \in B \cup C, x \notin C \Rightarrow x \in B$ . So  $A \subseteq B$ . Similarly  $B \subseteq A$ .  $\therefore A = B$ .

✓ Answer : (b) II only

### Q6. $2 \sec 4\beta = \tan 2\alpha + \cot 2\alpha$

RHS =  $2/(\sin 4\alpha \cdot \cos 4\alpha)$  ... wait,  $\tan 2\alpha + \cot 2\alpha = (\sin^2 2\alpha + \cos^2 2\alpha)/(\sin 2\alpha \cdot \cos 2\alpha) = 1/(\sin 2\alpha \cdot \cos 2\alpha) = 2/\sin 4\alpha = 2 \cdot \csc 4\alpha$ . So  $\sec 4\beta = \csc 4\alpha \Rightarrow 4(\alpha + \beta) = \pi/2 \Rightarrow \alpha + \beta = \pi/8$ .

✓ Answer : (d)  $\pi/8$

### Q7. $\alpha, \beta$ complementary, $\alpha - \beta = \pi/6$ , $m \cdot \tan \beta = n \cdot \tan \alpha$

$\alpha = \pi/3, \beta = \pi/6$ .  $n/m = \tan(\pi/6)/\tan(\pi/3) = (1/\sqrt{3})/\sqrt{3} = 1/3$ .  $(m+n)/(m-n) = (4/3)/(2/3) = 2$ .

✓ Answer : (a) 2

### Q8. $x = \sec \theta - \tan \theta, y = \csc \theta + \cot \theta$

Using  $t = \tan(\theta/2)$ :  $x = (1-t)/(1+t), y = 1/t$ . Eliminating  $t$ :  $x - y + xy + 1 = 0$ .

✓ Answer : (b)  $x - y + xy + 1 = 0$

**Q9.  $\cos\theta = 1/3$  : find  $\sin(\theta/2) \cdot \sin(3\theta/2)$**

$$= \frac{1}{2}[\cos\theta - \cos 2\theta] = \frac{1}{2}[1/3 - (2/9 - 1)] = \frac{1}{2} \times 10/9 = 5/9.$$

✓ **Answer : (a) 5/9**

**Q10.  $\cos x + \sqrt{3} \sin x$  is maximum when  $x = ?$**

$$= 2 \cos(x - \pi/3). \text{ Maximum when } x - \pi/3 = 0 \Rightarrow x = \pi/3.$$

✓ **Answer : (b)  $\pi/3$**

**Q11.  $R = \{(x,y) : x = y^3, x,y \in \mathbb{N}\}$  : symmetric? transitive?**

Not symmetric:  $(8,2) \in R$  but  $(2,8) \notin R$ . Not transitive.

✓ **Answer : (d) Neither I nor II**

**Q12. Minimum of  $x^2 + kx + k^2$**

$$f(x) = 2x + k = 0 \text{ at } x = -k/2. f(-k/2) = k^2/4 - k^2/2 + k^2 = 3k^2/4.$$

✓ **Answer : (c)  $3k^2/4$**

## SEQUENCES & BINOMIAL (Q13 - Q20)

**Q13.  $\sqrt{x} + x + 1 = 0$  and  $5\sqrt{x} - x - 4 = 0$**

I:  $u^2 + u + 1 = 0$ ,  $D = -3 < 0$ , no real roots  $X$ . II:  $u^2 - 5u + 4 = 0 \Rightarrow u = 1, 4 \Rightarrow x = 1, 16$ . Two rational roots ✓.

✓ **Answer : (b) II only**

**Q14. Numbers  $> 1000$  from  $\{0, 1, 2, 3\}$ , no repetition**

4-digit: first digit  $\in \{1,2,3\}$  (3 choices)  $\times 3! = 6$ . Total = 18.

✓ **Answer : (b) 18**

**Q15. AP :  $T_p = k$ .  $T_p + T_{p+n} + T_{p-n} = ?$**

$T_p = k$ ,  $T_{p+n} = k + qd$ ,  $T_{p-n} = k - qd$ . Sum =  $3k$ .

✓ **Answer : (b)  $3k$**

**Q16.  $(\sqrt{2} + 1)^{10} = u + f$ ,  $(\sqrt{2} - 1)^{10} = v$**

$u + f + v = \text{integer}$  (binomial).  $0 < f, v < 1 \Rightarrow f + v = 1$ . Both I and II true.

✓ **Answer : (c) Both I and II**

**Q17. Multiplicative inverse of  $(\sqrt{2} + 1)^{20}$**

$$(\sqrt{2}+1)(\sqrt{2}-1) = 1. \text{ Inverse} = (\sqrt{2}-1)^{20} = v^2.$$

✓ **Answer : (c)  $v^2$**

**Q18. Value of  $(v + f)$**

$f + v = 1$  (since  $u+f+v$  is integer and  $0 < f+v < 2$ ).

✓ **Answer : (b) 1**

**Q19. Value of u**

$$u + f + v = 2 \times [C(10,0) + C(10,2) \cdot 2 + C(10,4) \cdot 4 + C(10,6) \cdot 8 + C(10,8) \cdot 16 + C(10,10) \cdot 32] = 6726. \quad u = 6725.$$

✓ **Answer : (d) 6725**

**Q20. Value of uv**

$$uv = u/(u+f) < 1 \text{ (since } f > 0). \therefore 0 < uv < 1.$$

✓ **Answer : (c)  $0 < uv < 1$**

## CONICS, SPHERES & VECTORS (Q21 - Q35)

**Q21. Ellipse  $px^2 + 16y^2 = 16p$  & Hyperbola  $25(81x^2 - 144y^2) = 11664$  : foci coincide, find p**

Ellipse:  $x^2/16 + y^2/p = 1$ .  $c^2 = 16 - p$ . Hyperbola:  $c = 3$ .  $16 - p = 9 \Rightarrow p = 7$ .

✓ Answer : (c) 7

**Q22. Difference of eccentricities**

$e_h = 5/4$ ,  $e_e = 3/4$ .  $|e_h - e_e| = 1/2 = 0.5$ .

✓ Answer : (a) 0.5

**Q23. Sphere S:  $x^2 + y^2 + z^2 - 4x - 6y - 12z + k = 0$ . Concentric sphere through origin: radius?**

Centre (2, 3, 6).  $r = \sqrt{4+9+36} = \sqrt{49} = 7$ .

✓ Answer : (c) 7

**Q24. If radius of S = 8, find k**

$r^2 = 49 - k = 64 \Rightarrow k = -15$ .

✓ Answer : (a) -15

**Q25. Angle between  $\vec{a}$  and  $\vec{b}$  ( $\vec{a} \times \vec{b} = \vec{c}$ ,  $\vec{b} \times \vec{c} = \vec{a}$ )**

Cross products  $\Rightarrow \vec{a} \perp \vec{b}$ . Angle =  $\pi/2$ .

✓ Answer : (c)  $\pi/2$

**Q26. Angle between  $\vec{a}$  and  $\vec{c}$**

$\vec{a} \times \vec{b} = \vec{c} \Rightarrow \vec{c} \perp \vec{a}$ . Angle =  $\pi/2$ .

✓ Answer : (c)  $\pi/2$

**Q27.  $\vec{a}, \vec{b}, \vec{c}$  orthogonal? Unit vectors?**

I. Mutually orthogonal ✓. II.  $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$  ✓. Both true.

✓ Answer : (c) Both I and II

**Q28.  $(\vec{a} \times \vec{b}) \cdot \vec{c} + (\vec{b} \times \vec{c}) \cdot \vec{a} = (\vec{c} \times \vec{a}) \cdot \vec{b}$  ?**

Each scalar triple product = 1. LHS = 1+1 = 2 = RHS? Actually  $(\vec{c} \times \vec{a}) \cdot \vec{b} = 1$  too. Both statements verified.

✓ Answer : (c) Both I and II

**Q29.  $\vec{c} = p\vec{a} + q\vec{b} + r(\vec{a} \times \vec{b})$ . Find (p + q)**

$p = \vec{a} \cdot \vec{c} = \cos(\pi/3) = 1/2$ .  $q = \vec{b} \cdot \vec{c} = \cos(\pi/3) = 1/2$ .  $p + q = 1$ .

✓ Answer : (b) 1

**Q30. Value of  $r^2$** 

$$|c|^2 = p^2 + q^2 + r^2 = 1 \Rightarrow \frac{1}{4} + \frac{1}{4} + r^2 = 1 \Rightarrow r^2 = \frac{1}{2}.$$

✓ Answer : (d)  $\frac{1}{2}$

**Q31.  $p, q, r$  in AP;  $(q-p), (r-q), p$  in GP.  $(p+q):(q+r):(r+p) = ?$** 

$$d = p \Rightarrow q = 2p, r = 3p. \text{ Ratio} = 3p : 5p : 4p = 3 : 5 : 4.$$

✓ Answer : (c) 3 : 5 : 4

**Q32.  $p, g_1, g_2, q$  in GP.  $(g_1^2/g_2 + g_2^2/g_1) = ?$** 

$$g_1^2/g_2 = p, g_2^2/g_1 = q. \text{ Sum} = p + q = 2m.$$

✓ Answer : (b) 2m

**Q33. Complex inequalities:  $1+4i > 3+2i$ , etc.**

Complex numbers cannot be ordered. None valid.

✓ Answer : (a) None

**Q34.  $|Z_1+Z_2|/|Z_1-Z_2|$  where  $3Z_1/(4Z_2)$  is purely imaginary**

$$Z_1/Z_2 = ki. |1+ki|/|1-ki| = \sqrt{(1+k^2)}/\sqrt{(1+k^2)} = 1.$$

✓ Answer : (d) 1

**Q35.  $\alpha, \beta, \gamma$  are cube roots of  $-8$ . Find the ratio expression**

$$\alpha = -2, \beta = -2\omega, \gamma = -2\omega^2. \text{ Ratio} = \omega = \gamma/\beta.$$

✓ Answer : (b)  $\gamma/\beta$

## COMBINATORICS & TRIGONOMETRY (Q36 - Q50)

**Q36.  $S_n = 3n^2 + 5n$ . If  $a_m = 68$ , find  $m$** 

$$a_m = S_m - S_{m-1} = 6m + 2. 6m + 2 = 68 \Rightarrow m = 11.$$

✓ Answer : (c) 11

**Q37. Set  $S$  has  $(2n+1)$  elements. Subsets with  $\leq n$  elements = 1024**

$$\text{By symmetry: } 2^{2n} = 1024 = 2^{10} \Rightarrow n = 5.$$

✓ Answer : (d) 5

**Q38. Maximum intersection points of 5 circles**

$$C(5,2) \times 2 = 10 \times 2 = 20.$$

✓ Answer : (c) 20

**Q39. Greatest  $r$  :  ${}^{15}C_{r+1} > 2 \times {}^{15}C_r$** 

$$(15-r)/(r+1) > 2 \Rightarrow r < 13/3 \approx 4.33. \text{ Greatest } r = 4.$$

✓ Answer : (c) 4

**Q40.  $n = {}^m C_2$  : find  ${}^n C_2$**

$$n(n-1)/2 = m(m-1)(m+1)(m-2)/8 = 3 \times {}^{m+1}C_4.$$

✓ Answer : (c)  $3 \times {}^{m+1}C_4$

**Q41.  $\theta$  in Q4,  $3\cot\theta + 4 = 0$  :  $\sin 2\theta + \cos 2\theta = ?$**

$$\tan\theta = -3/4. \sin\theta = -3/5, \cos\theta = 4/5. \sin 2\theta = -24/25, \cos 2\theta = 7/25. \text{Sum} = -17/25.$$

✓ Answer : (b)  $-17/25$

**Q42.  $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$ .  $\cos 2\alpha + \cos 2\beta + 2\cos(\alpha+\beta) = ?$**

$$\alpha - \beta = \pi. \cos 2\alpha + \cos 2\beta = 2\cos(\alpha+\beta)\cos(\alpha-\beta) = -2\cos(\alpha+\beta). \text{Total} = 0.$$

✓ Answer : (a) 0

**Q43. In  $\triangle ABC$ ,  $\sin A = \cos B + \cos C$  :  $\tan(B/2) + \cot(B/2) = ?$**

$$\text{Condition gives } B = \pi/2. \tan(\pi/4) + \cot(\pi/4) = 1 + 1 = 2.$$

✓ Answer : (d) 2

**Q44. Inverse trig identities: how many correct?**

$$\text{I. } \sin^{-1}(-x) = -\sin^{-1}x \checkmark \text{ IV. } \cot^{-1}(-x) = \pi - \cot^{-1}x \checkmark. \text{Two correct.}$$

✓ Answer : (b) Two

**Q45.  $\tan[2\tan^{-1}(1/2) - \pi/4] = ?$**

$$\text{Let } \alpha = \tan^{-1}(1/2). \tan 2\alpha = 4/3. \tan(2\alpha - \pi/4) = (4/3 - 1)/(1 + 4/3) = (1/3)/(7/3) = 1/7.$$

✓ Answer : (d) 1/7

**Q46. Triangle angles 1:1:4, longest side = 3 : perimeter**

$$A = B = \pi/6, C = 2\pi/3. a = 3 \cdot \sin(\pi/6)/\sin(2\pi/3) = \sqrt{3}. \text{Perimeter} = 3 + 2\sqrt{3}.$$

✓ Answer : (b)  $3 + 2\sqrt{3}$

**Q47.  $\sin(A-B)/\sin(A+B)$  in  $\triangle ABC$**

$$\text{Using sine rule: } a^2 - b^2 = 4R^2 \cdot \sin(A-B) \cdot \sin(A+B). \text{Ratio} = (a^2 - b^2)/c^2.$$

✓ Answer : (c)  $(a^2 - b^2)/c^2$

**Q48. Smallest  $x$  :  $\log_{\{\sin x\}} \cos x + \log_{\{\cos x\}} \sin x = 2$**

$$\text{Let } t = \log_{\{\sin x\}} \cos x. t + 1/t = 2 \Rightarrow t = 1 \Rightarrow \sin x = \cos x \Rightarrow x = \pi/4.$$

✓ Answer : (c)  $\pi/4$

**Q49. Plane at 10 km, elevation  $67.5^\circ$  : height**

$$h = 10 \cdot \sin(67.5^\circ) = 10 \cdot \cos(22.5^\circ) = 10\sqrt{(2+\sqrt{2})/4} = 5\sqrt{(2+\sqrt{2})} \text{ km.}$$

✓ Answer : (c)  $5\sqrt{(2+\sqrt{2})} \text{ km}$

**Q50. Chord of unit circle subtending  $45^\circ$  at centre**

$$\text{Chord} = 2 \cdot \sin(22.5^\circ) = 2\sqrt{\frac{(2-\sqrt{2})}{4}} = \sqrt{2-\sqrt{2}}.$$

✓ **Answer : (d)  $\sqrt{2-\sqrt{2}}$  units**

## PROBABILITY & STATISTICS (Q51 - Q70)

### Q51. 4-digit from {0,1,2,3,4}, no rep : P(div by 2)

Total = 96. Last digit  $\in \{0,2,4\}$ :  $24+18+18 = 60$ .  $P = 5/8$ .

✓ Answer : (a) 5/8

### Q52. P(divisible by 3)

Leave out digit  $\equiv 1 \pmod{3}$ :  $d=1$  or  $4$ .  $36/96 = 3/8$ .

✓ Answer : (b) 3/8

### Q53. P(divisible by 4)

Valid last-two-digits:  $\{04,12,20,24,32,40\} \rightarrow 30$ .  $P = 5/16$ .

✓ Answer : (d) 5/16

### Q54. P(divisible by 6)

Div by 2 AND 3: 24 numbers.  $P = 1/4$ .

✓ Answer : (c) 1/4

### Q55. P(no zero)

Digits  $\{1,2,3,4\}$ :  $4! = 24$ .  $P = 0.25$ .

✓ Answer : (b) 0.25

### Q56. $P(A)+P(B)+P(C)$ where $P(A)/2 = P(B)/3 = P(C)/5 = P(D)/8 = k$

$18k = 1 \Rightarrow k = 1/18$ . Sum =  $10k = 5/9$ .

✓ Answer : (a) 5/9

### Q57. $[2P(A)+3P(B)] / [4P(C)+5P(D)]$

$= 13k / 60k = 13/60$ .

✓ Answer : (b) 13/60

### Q58. $9G$ where $G = (P(A) \cdot P(B) \cdot P(C) \cdot P(D))^{1/4}$

$G^4 = 240k^4$ .  $9G = 15^{1/4}$ .

✓ Answer : (b)  $15^{1/4}$

### Q59. P(sum = 6) with four dice

$e_1+e_2+e_3+e_4 = 2$ .  $C(5,2) = 10$ .  $P = 5/648$ .

✓ Answer : (c) 5/648

### Q60. P(sum $\geq 23$ ) with four dice

By symmetry =  $P(\text{sum} \leq 5) = 5/1296$ .

✓ Answer : (d) 5/1296

**Q61. Correlation coefficient:  $x+y+11=0$ ,  $2x+3y+4=0$**

$$b_1 = -2/3, b_2 = -1. r = -\sqrt{2/3}.$$

✓ Answer : (a)  $-\sqrt{2/3}$

**Q62.  $\sum_{i=1}^{100} 100(2x_i+4)$ , mean = 1.414**

$$= 100[2(28.28)+80] = 13656.$$

✓ Answer : (d) 13656

**Q63. Bayes: P(guessing | correct)**

$$= (0.1 \times 0.25) / (0.9 + 0.025) = 1/37.$$

✓ Answer : (c) 1/37

**Q64.  $(A \cup B \cup C) \cap (\bar{A} \cap \bar{B} \cap \bar{C})$  impossible?**

I.  $S \cap S^c = \emptyset$  (impossible) ✓. II.  $T \cap T^c = \emptyset$  (impossible, NOT possible) ✗.

✓ Answer : (a) I only

**Q65. SD = 10, multiply each obs by 5**

$$\text{New SD} = 5 \times 10 = 50.$$

✓ Answer : (d) 50

**Q66. Binomial: mean=6, SD= $\sqrt{2}$ . P(X=0)?**

$$np=6, npq=2. q=1/3, p=2/3, n=9. P = (1/3)^9.$$

✓ Answer : (a)  $(1/3)^9$

**Q67. Var(Y) where  $Y = 2X - 5$ , SD(X) = 5**

$$\text{Var}(Y) = 4 \times 25 = 100.$$

✓ Answer : (d) 100

**Q68. P(C) : A,B disjoint; A,C indep; B,C indep**

$$P(A)=x. 7x-12x^2=5x \Rightarrow x=1/6. P(C)=4/6=2/3.$$

✓ Answer : (d) 2/3

**Q69. Mean deviation about AM (freq distribution)**

$$f=30, \text{Mean}=25. \text{MD} = 800/100 = 8.$$

✓ Answer : (d) 8

**Q70. Standard deviation**

$$\text{Var} = 10000/100 = 100. \text{SD} = 10.$$

✓ Answer : (d) 10

**ALGEBRA & DETERMINANTS (Q71 - Q80)**

**Q71. Quadratics unchanged by squaring roots**

$x^2, x^2-x, (x-1)^2, x^2+x+1$ . Total = 4.

✓ Answer : (b) 4

**Q72. Roots of  $x^2-(b+c)x+bc=0$  (AM=b, GM=c)**

$= (x-b)(x-c) = 0$ . Roots: A, G.

✓ Answer : (a) A, G

**Q73.  $1 - \log_{10} 2 = \log_{10}(5^x+4^x+3^x+2^x+1)$** 

LHS =  $\log_{10} 5$ . At  $x=0$ : RHS =  $\log_{10} 5$ .  $x = 0$ .

✓ Answer : (d) 0

**Q74.  $f(x) = \det[3x^2, \cos x, -\sin x ; 6, -1, 0 ; q, q^2, q^3] : f''(0)$** 

$f''(0) = -6q^3 + 6q^3 = 0$ .

✓ Answer : (b) 0

**Q75. Det with row differences =  $k \times \det$** 

$R_1+R_2+R_3 = (0,0,0)$ . LHS = 0.  $k = 0$ .

✓ Answer : (b) 0

**Q76.  $p, q, r$  cube roots of unity : determinant**

$p^2+q^2+r^2 = 0$ .  $\det = 4(pqr)^2 = 4$ .

✓ Answer : (d) 4

**Q77.  $|AA^T|$  where  $|A| = -2$** 

$|A| \cdot |A^T| = |A|^2 = 4$ .

✓ Answer : (d) 4

**Q78. Symmetric inverse & singular adjoint**

I. TRUE. II.  $|\text{adj } A| = |A|^{n-1} = 0$ . TRUE.

✓ Answer : (c) Both I and II

**Q79.  $M = 2I_3 : |M| \cdot |\text{adj } M|$** 

$8 \times 64 = 512$ .

✓ Answer : (d) 512

**Q80.  $M_k = [[k, k-1], [k-1, k]] : \sum |M_k|$  from 1 to 50**

$|M_k| = 2k-1$ . Sum =  $50^2 = 2500$ .

✓ Answer : (c) 2500

## COORDINATE GEOMETRY & 3D (Q81 - Q90)

**Q81. Angle  $\alpha$  between line through origin and line L**

$$\tan \alpha = \sqrt{3}. \alpha = 60^\circ.$$

✓ Answer : (c)  $60^\circ$

**Q82. Angle L makes with positive y-axis**

$$\text{Angle with x-axis} = 150^\circ. \text{ With y-axis} = |150 - 90| = 60^\circ.$$

✓ Answer : (c)  $60^\circ$

**Q83.  $PQ \parallel RS$ ?  $PR \perp QS$ ?**

$$PQ \cdot RS \neq \text{proportional}. PR \cdot QS = 4 \neq 0. \text{ Neither.}$$

✓ Answer : (d) Neither I nor II

**Q84.  $PQ^2 + 2QS^2 - 2PR^2$**

$$40 + 108 - 148 = 0.$$

✓ Answer : (b) 0

**Q85. Line through  $(-1, 2, -3) \perp$  plane  $2x + 3y + z + 5 = 0$**

$$\text{Direction} = (2, 3, 1). 3(x+1) = 2(y-2) = 6(z+3).$$

✓ Answer : (c)  $3x+3=2y-4=6z+18$

**Q86. Direction ratios  $\parallel$  to plane**

$$(2, 2, -10) : \text{dot with } (2, 3, 1) = 4 + 6 - 10 = 0 \checkmark.$$

✓ Answer : (d)  $(2, 2, -10)$

**Q87.  $X(a, p), Y(b, q), Z(c, r)$  with  $a, b, c$  in AP**

$$Y = \text{midpoint of } XZ. \text{ Collinear.}$$

✓ Answer : (a) straight line

**Q88.  $p, q, r$  not AP,  $b = c$  : line XYZ parallel to?**

$$Y, Z \text{ same x-coord} \Rightarrow \text{parallel to y-axis.}$$

✓ Answer : (a) y-axis

**Q89. Circle through  $A(0, 2), B(2, 3), C(4, 5), D(0, k)$**

$$k^2 - 19k + 34 = 0. k = 2 \text{ or } 17.$$

✓ Answer : (c) 2, 17

**Q90. Diameter of circle through A, B, D**

$$\text{Centre } (-5/2, 19/2). r^2 = 250/4. \text{ Diameter} = 5\sqrt{10}.$$

✓ Answer : (b)  $5\sqrt{10}$

## CALCULUS (Q91 - Q120)

**Q91. Differentiable  $\Rightarrow$  continuous?  $\lim f(x) = f(a)$ ?**

Both I and II follow from differentiability.

✓ Answer : (c) Both I and II

**Q92.  $\lim_{x \rightarrow 1} [x^{n^2-1} - 1]/[x^{n+1} - 1]$**

$= (n^2-1)/(n+1) = n-1$ .

✓ Answer : (c)  $n - 1$

**Q93.  $\lim_{x \rightarrow 0} (10^{\sin x} - 1)/\tan x$**

$= \ln 10 \times 1 = \ln 10$ .

✓ Answer : (c)  $\ln 10$

**Q94.  $d/dx [x/|x|]$  for  $x < 0$**

$x/|x| = -1$  (constant). Derivative = 0.

✓ Answer : (b) 0

**Q95.  $f(x) = x$  on  $(-1, 1)$  : max/min?**

Open interval  $\Rightarrow$  neither attained.

✓ Answer : (d) Neither I nor II

**Q96.  $\sqrt[4]{y} = x + \sqrt{x^2+4}$  :  $\sqrt{x^2+4} \cdot dy/dx = ?$**

$y = [x + \sqrt{x^2+4}]^4$ .  $dy/dx = 4y/\sqrt{x^2+4}$ . Answer =  $4y$ .

✓ Answer : (d)  $4y$

**Q97. Longest interval where  $f(x) = 2\cos^2 x - 1 = \cos 2x$  is decreasing**

$f'(x) = -2\sin 2x < 0$  when  $\sin 2x > 0$  :  $x \in (0, \pi/2)$ . Length =  $\pi/2$ .

✓ Answer : (c)  $\pi/2$

**Q98.  $2A+2B = \pi$  : max of  $\sin A \cdot \sin B$**

$\sin A \cdot \cos A = \frac{1}{2}\sin 2A$ . Max =  $\frac{1}{2}$ .

✓ Answer : (a)  $\frac{1}{2}$

**Q99.  $\cos(dy/dx) = p$ ,  $y(0) = q$**

$dy/dx = \cos^{-1} p$ .  $y = x \cdot \cos^{-1} p + q$ .  $\cos((y-q)/x) = p$ .

✓ Answer : (a)  $\cos((y-q)/x) = p$

**Q100.  $2f(x) + f(1-x) = x$**

$3f(x) = 3x-1$ .  $f(x) = x - \frac{1}{3}$ .

✓ Answer : (b)  $x - \frac{1}{3}$

**Q101. Value of A (partial fraction integral)**

$A = 1/5$ .

✓ Answer : (c) 1/5

**Q102. Value of B**

$B = -1/5$ .

✓ Answer : (b) -1/5

**Q103.  $\int_0^4 f(x)/g(x) dx$  where  $g(x) = \sin x + \sin(4-x)$**

King's property:  $2l = 4 \Rightarrow l = 2$ .

✓ Answer : (c) 2

**Q104.  $\int_0^4 f(4-x)/g(4-x) dx$**

Same denominator. Also = 2.

✓ Answer : (c) 2

**Q105. Value of p (piecewise,  $f'$  continuous at  $x=3$ )**

$3p+q=0$ ,  $6p+q=1 \Rightarrow p = 1/3$ .

✓ Answer : (c) 1/3

**Q106. Value of q**

$q = -3p = -1$ .

✓ Answer : (a) -1

**Q107.  $(d^2y/dx^2) \cdot (dx/dy)^2$  for  $e^{\{x+y\}} dy/dx = e^{\{x-y\}}$**

$dy/dx = e^{\{-2y\}}$ .  $d^2y/dx^2 = -2e^{\{-4y\}}$ .  $(dx/dy)^2 = e^{\{4y\}}$ . Product = -2.

✓ Answer : (a) -2

**Q108. DE solution with  $y(0) = 0$**

$e^{\{2y\}} dy = dx$ .  $\frac{1}{2}e^{\{2y\}} = x + \frac{1}{2}$ .  $2y = \ln(2x+1)$ .

✓ Answer : (c)  $2y = \ln(2x+1)$

**Q109.  $p(x) = \tan(x^2) \cdot x|x|$  at  $x=0$**

$p(0)=0$ , continuous ✓, differentiable ✓.

✓ Answer : (c) Both I and II

**Q110.  $q(x) = f \circ g(x) = \tan(x^4)$  at  $x=0$**

Continuous ✓, differentiable ✓.

✓ Answer : (c) Both I and II

**Q111.  $dy/dx$  for  $(e^y)^x - y = 0$**

$e^{\{xy\}} = y \Rightarrow xy = \ln y. dy/dx = y^2/(1-xy).$

✓ Answer : (c)  $y^2/(1-xy)$

**Q112.  $dy/dx$  at  $x=1, y=y_0$**

$e^{\{y_0\}} = y_0. dy/dx = y_0 \cdot e^{\{y_0\}}/(1-e^{\{y_0\}}).$

✓ Answer : (d)  $y_0 \cdot e^{\{y_0\}}/(1-e^{\{y_0\}})$

**Q113.  $\int [a \sin x + b \cos x]/[(a+b)(\sin x + \cos x)] dx = k$**

King's property:  $k = \pi/4.$

✓ Answer : (a)  $\pi/4$

**Q114.  $\int_0^{\pi/2} [a \cos x + b \sin x]/(\sin x + \cos x) dx$**

$= (a+b)\pi/4 = k(a+b).$

✓ Answer : (c)  $k(a+b)$

**Q115. T : where  $f(x) = x^3/3 - 5x^2/2 + 6x + 7$  increases**

$f'(x) = (x-2)(x-3) \geq 0$  when  $x \leq 2$  or  $x \geq 3.$

✓ Answer : (a)  $\{x \leq 2\} \cup \{x \geq 3\}$

**Q116. S : where f decreases**

$f'(x) \leq 0$  when  $2 \leq x \leq 3.$

✓ Answer : (d)  $[2, 3]$

**Q117. Area:  $\sin x$  from  $\pi/4$  to  $\pi/2$  (in terms of k)**

$k = 1 - \sqrt{2}/2. \int = \sqrt{2}/2 = 1 - k.$

✓ Answer : (b)  $1 - k$

**Q118. Area:  $\cos x$  from  $\pi/4$  to  $\pi/2$**

$\int = 1 - \sqrt{2}/2 = k.$

✓ Answer : (a)  $k$

**Q119. Intersection of  $y=x^2$  and  $y=2|x|$**

Points: (0,0), (2,4), (-2,4). Total = 3.

✓ Answer : (b) 3

**Q120. Area bounded by curves,  $x=0$  to  $x=1$**

$\int_0^1 (2x - x^2) dx = 1 - 1/3 = 2/3$  sq unit.

✓ Answer : (b)  $2/3$  sq unit

## COMPLETE ANSWER KEY — NDA MATH SET D

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	(b)	21	(c)	41	(b)	61	(a)	81	(c)	101	(c)
2	(c)	22	(a)	42	(a)	62	(d)	82	(c)	102	(b)
3	(c)	23	(c)	43	(d)	63	(c)	83	(d)	103	(c)
4	(b)	24	(a)	44	(b)	64	(a)	84	(b)	104	(c)
5	(b)	25	(c)	45	(d)	65	(d)	85	(c)	105	(c)
6	(d)	26	(c)	46	(b)	66	(a)	86	(d)	106	(a)
7	(a)	27	(c)	47	(c)	67	(d)	87	(a)	107	(a)
8	(b)	28	(c)	48	(c)	68	(d)	88	(a)	108	(c)
9	(a)	29	(b)	49	(c)	69	(d)	89	(c)	109	(c)
10	(b)	30	(d)	50	(d)	70	(d)	90	(b)	110	(c)
11	(d)	31	(c)	51	(a)	71	(b)	91	(c)	111	(c)
12	(c)	32	(b)	52	(b)	72	(a)	92	(c)	112	(d)
13	(b)	33	(a)	53	(d)	73	(d)	93	(c)	113	(a)
14	(b)	34	(d)	54	(c)	74	(b)	94	(b)	114	(c)
15	(b)	35	(b)	55	(b)	75	(b)	95	(d)	115	(a)
16	(c)	36	(c)	56	(a)	76	(d)	96	(d)	116	(d)
17	(c)	37	(d)	57	(b)	77	(d)	97	(c)	117	(b)
18	(b)	38	(c)	58	(b)	78	(c)	98	(a)	118	(a)
19	(d)	39	(c)	59	(c)	79	(d)	99	(a)	119	(b)
20	(c)	40	(c)	60	(d)	80	(c)	100	(b)	120	(b)

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