1. If $f(x)$ is a function satisfying $f(x+y) = f(x)f(y), \forall x, y \in N$ such that $f(1) = 3$ and $\sum_{x=1}^{n} f(x) = 120$. Then, the value of n is
(a) 4
(b) 5
(c) 6
(d) None of these

2. Find the domain and range of the relation R defined by

$$R = \{(x, x+5): x \in \{0,1,2,3,4,5\}\}$$

(a) $\{0,1,2,3,4,5\}, \{5,6,7,8,9,10\}$
(b) $\{1,2,3,4,5\}, \{5,6,7,8,9,10\}$
(c) $\{0,1,2,3,4,5\}, \{6,7,8,9,10\}$
(d) None of the above

3. Let f and g be two real functions defined by $f(x) = \frac{1}{x+4}, x \neq -4$ and $g(x) = (x+4)^3$, then f/g is equal to
(a) $(x+4)^2$
(b) $\frac{1}{(x+4)^4}$
(c) $\frac{1}{(x+4)^3}$
(d) $\frac{1}{(x+4)^2}$

4. Find the range of $f(x) = 1 + 3\cos 2x$.
(a) [2,3]
(b) [2,4]
(c) $[-2,4]$
(d) None of the above

5. Find the domain of the function $f(x) = \frac{1}{\sqrt{x+|x|}}$.
(a) R
(b) R^+
(c) R^-
(d) $R + R$

6. The relation f is defined by $f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$ and the relation g is defined by $g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$. Which of the following relation is a function?
(a) f
(b) g
(c) f, g
(d) None of the above

7. Let $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}, x \in R - \{0\}$, then $f(x)$ is equal to
(a) x^2
(b) $x^2 - 1$
(c) $x^2 - 2$, when $|x| \geq 2$
(d) None of these

8. If $f: R \to R$ satisfies $f(x+y) = f(x) + f(y)$, $\forall x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is
(a) $\frac{7n}{2}$
(b) $\frac{7(n+1)}{2}$
(c) $7n(n+1)$
(d) $\frac{7n(n+1)}{2}$

9. Let R be the set of real numbers and the functions $f: R \to R$ and $g: R \to R$ be defined by $f(x) = x^2 + 2x - 3$ and $g(x) = x + 1$. Then, the value of x for which $g(f(x)) = f(g(x))$ is
(a) -1
(b) 0
(c) 1
(d) 2

10. The domain of the function $f(x) = \sqrt{\frac{(x+1)(x-3)}{x-2}}$ is
(a) $[-1,2) \cup [3, \infty)$
(b) $(-1,2) \cup [3, \infty)$
(c) $[-1,2] \cup [3, \infty)$
(d) None of these

11. The inverse of the function $y = [1 - (x-3)^4]^{1/7}$ is
(a) $3 + (1 - x^7)^{1/4}$
(b) $3 - (1 - x^7)^{1/4}$
(c) $3 - (1 + x^7)^{1/4}$
(d) None of these

12. Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$, where the function f satisfies $f(x+y) = f(x)f(y)$ for all natural numbers x, y and $f(1) = 2$. Then, the natural number ' a ' is
(a) 2
(b) 4
(c) 3
(d) 16

13. If the function $f(x) = \begin{cases} \frac{x^2}{a}, & 0 \leq x < 1 \\ a, & 1 \leq x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^2}, & \sqrt{2} \leq x < \infty \end{cases}$ is continuous for $0 \leq x < \infty$, then the most suitable values of a and b are
(a) $a = 1, b = -1$

(b) $a = -1, b = 1 + \sqrt{2}$
(c) $a = -1, b = 1$
(d) None of these

14. Let $f(x) = a + b|x| + c|x|^4$, where a, b and c are real constants. Then, $f(x)$ is differentiable at $x = 0$, if
(a) $a = 0$
(b) $b = 0$
(c) $c = 0$
(d) None of these

15. If $x = a(\theta + \sin\ \theta)$ and $y = a(1 - \cos\ \theta)$, then dy/dx is equal to
(a) $\tan\ \frac{\theta}{2}$
(b) $\cot\ \frac{\theta}{2}$
(c) $\tan\ \theta$
(d) $\cot\ \theta$

16. If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin\ x^2$, then $\frac{dy}{dx}$ is equal to
(a) $\sin\ \left(\frac{2x-1}{x^2+1}\right)^2 \left[\frac{2+2x+x^2}{(x^2+1)^2}\right]$
(b) $\sin\ \left(\frac{2x-1}{x^2+1}\right)^2 \left[\frac{2+2x-2x^2}{(x^2+1)^2}\right]$
(c) $\sin\ \left(\frac{2x-1}{x^2+1}\right)^2 \left[\frac{2+2x-x^2}{(x^2+1)^2}\right]$
(d) None of the above

17. If $x = a(\theta + \sin\ \theta)$ and $y = a(1 - \cos\ \theta)$, then dy/dx is equal to
(a) $\tan\ \frac{\theta}{2}$
(b) $\cot\ \frac{\theta}{2}$
(c) $\tan\ \theta$
(d) $\cot\ \theta$

18. If $x = \sqrt{a^{\sin^{-1}\ t}}$ and $y = \sqrt{a^{\cos^{-1}\ t}}$, then the value of $\frac{dy}{dx}$ is
(a) $\frac{y}{x}$
(b) $\frac{x}{y}$
(c) $-\frac{y}{x}$
(d) $-\frac{x}{y}$

19. If $y = x - x^2$, then the derivative of y^2 w.r.t. x^2 is
(a) $2x^2 + 3x - 1$
(b) $2x^2 - 3x + 1$
(c) $2x^2 + 3x + 1$
(d) None of these

20. If $y = x + e^x$, then $\frac{d^2x}{dy^2}$ is equal to
(a) $\frac{1}{(1+e^x)^2}$
(b) $-\frac{e^x}{(1+e^x)^2}$
(c) $-\frac{e^x}{(1+e^x)^3}$
(d) e^x

21. If $8f(x) + 6f\left(\frac{1}{x}\right) = x + 5$ and $y = x^2 f(x)$ then $\frac{dy}{dx}$ at $x = -1$, is equal to
(a) 0
(b) $\frac{1}{14}$
(c) $-\frac{1}{14}$
(d) None of the above

22. Let f be a function defined for all $x \in R$. If f is differentiable and $f(x^3) = x^5, \forall x \in R, x \neq 0$, then the value of $f'(27)$ is
(a) 15
(b) 45
(c) 0
(d) None of the above

23. If $\frac{d}{dx}\left(\frac{1+x^4+x^8}{1+x^2+x^4}\right) = ax^3 + bx$, then
(a) $a = 4, b = 2$
(b) $a = 4, b = -2$
(c) $a = -2, b = 4$
(d) None of the above

24. Let $f(x) = (x^3 + 2)^{30}$. If $f^n(x)$ is a polynomial of degree 20, where $f^n(x)$ denotes the nth derivative of $f(x)$ w.r.t. x, then the value of n is
(a) 60
(b) 40
(c) 70
(d) None of these

25. If $\sin\ y = x\sin\ (a + y)$, then $\frac{dy}{dx}$ is equal to
(a) $\frac{\sin\ a}{\sin^2\ (a+y)}$
(b) $\frac{\sin^2\ (a+y)}{\sin\ a}$
(c) $\sin\ a\sin^2\ (a + y)$
(d) $\frac{\sin^2\ (a-y)}{\sin\ a}$

26. If $y = f(x^2 + 2)$ and $f'(3) = 5$, then $\frac{dy}{dx}$ at $x = 1$ is
(a) 5
(b) 25

(c) 15
(d) 10

27. The minimum value of $\frac{x}{\log x}$ is
(a) e
(b) $1/e$
(c) 1
(d) None of these

28. If $f(x) = |x| + |x-1| + |x-2|$, then
(a) $f(x)$ has minima at $x = 1$
(b) $f(x)$ has maxima at $x = 1$
(c) $f(x)$ has neither maxima nor minima at $x = 1$
(d) None of the above

29. The greatest value of $f(x) = (x+1)^{1/3} - (x-1)^{1/3}$ on [0,1] is
(a) 1
(b) 2
(c) 3
(d) $\frac{1}{3}$

30. The minimum value of $2\log_{10} x - \log_x 0.01, x > 1$, is
(a) 1
(b) -1
(c) 2
(d) None of the above

31. If $y = a\log_e x + bx^2 + x$ has its extreme values (i.e. maximum or minimum value) at $x = 1$ and $x = 2$, then the values of a and b are
(a) $a = -\frac{1}{6}, b = \frac{4}{3}$
(b) $a = -\frac{4}{3}, b = \frac{1}{6}$
(c) $a = \frac{4}{3}, b = -\frac{1}{6}$
(d) None of these

32. If $y = x^2 + ax + b$ has a minimum at $x = 3$ and the minimum value is 5 , then the values of a and b are
(a) $a = 6, b = -14$
(b) $a = -6, b = 14$
(c) $a = 14, b = -14$
(d) $a = -14, b = 6$

33. The greatest value of the function $f(x) = xe^{-x}$ in $[0, \infty)$ is
(a) 0
(b) $\frac{1}{e}$
(c) $-e$
(d) None of these

34. The least value of the function $f(x) = \cos^{-1} x^2$ in the interval $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ is
(a) $\frac{\pi}{3}$
(b) $\frac{\pi}{2}$
(c) $-\frac{\pi}{3}$
(d) None of these

35. $\int \frac{2^x+3^x}{5^x} dx$ is equal to
(a) $\frac{2^x}{\log 2} + \frac{3^x}{\log 3} - \frac{5^x}{\log 5} + C$
(b) $\frac{2/5}{x\log_e (2/5)} + \frac{3/5}{x\log_e (3/5)} + C$
(c) $\frac{\left(\frac{2}{5}\right)^x}{\log_e \left(\frac{2}{5}\right)} + \frac{\left(\frac{3}{5}\right)^x}{\log_e \left(\frac{3}{5}\right)} + C$
(d) None of the above

36. If $\int \frac{2^{1/x}}{x^2} dx = a2^{1/x} + C$, then a is equal to
(a) $-\log_2 e$
(b) $-\log_e 2$
(c) -1
(d) $\frac{1}{2}$

37. $\int \frac{dx}{\sqrt{1+4x^2}}$ is equal to
(a) $\frac{1}{2}\log\left|x + \sqrt{2x^2+1}\right| + C$
(b) $\frac{1}{2}\log\left|2x + \sqrt{4x^2+1}\right| + C$
(c) $\log\left|2x + \sqrt{4x^2+1}\right| + C$
(d) None of the above

38. $\int \frac{1}{1+\cos^2 x} dx$ is equal to
(a) $\frac{1}{\sqrt{2}}\tan^{-1}(\tan x) + C$
(b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{2}\tan x\right) + C$
(c) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\tan x\right) + C$
(d) None of the above

39. $\int \frac{dx}{5+4\cos x}$ is equal to
(a) $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan x\right) + C$
(b) $\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\tan x\right) + C$
(c) $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan \frac{x}{2}\right) + C$
(d) $\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\tan \frac{x}{2}\right) + C$

40. $\int \frac{3\sin x + 2\cos x}{3\cos x + 2\sin x} dx$ is equal to
(a) $\frac{12}{13}x - \frac{5}{13}\log(3\cos x + 2\sin x) + C$
(b) $\frac{12}{13}x + \frac{5}{13}\log(3\cos x + 2\sin x) + C$

(c) $\frac{13}{12}x \times \frac{5}{13}\log(3\cos x + 2\sin x) + C$
(d) None of the above

41. $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx$ is equal to
(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{4}$
(c) $\frac{3\pi}{2}$
(d) π

42. $\int_2^8 |x-5|dx$ is equal to
(a) 9
(b) 3
(c) 6
(d) 15

43. $\int_0^2 [x^2]dx$ is equal to
(a) $2-\sqrt{2}$
(b) $2+\sqrt{2}$
(c) $\sqrt{2}-1$
(d) $-\sqrt{2}-\sqrt{3}+5$

44. $\int_0^3 [\sqrt{x}]dx$ is equal to
(a) 1
(b) 2
(c) -1
(d) -2

45. $\int_0^{\pi} xf(\sin x)dx$ is equal to
(a) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$
(b) $\frac{\pi}{4}\int_0^{\pi} f(\sin x)dx$
(c) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$
(d) $\frac{\pi}{2}\int_0^{\pi/2} f(\cos x)dx$

46. The value of the integral $\int_0^{\pi} \frac{1}{e^{\cos x}+1}dx$ is
(a) π
(b) 0
(c) 2π
(d) $\frac{\pi}{2}$

47. The value of $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)dx$ is
(a) 0
(b) 2
(c) π
(d) 1

48. $\int_{-\pi}^{\pi} \frac{e^{\sin x}}{e^{\sin x}+e^{-\sin x}}dx$ equals
(a) $\frac{\pi}{2}$
(b) 2π
(c) π
(d) $\frac{\pi}{4}$

49. The value of $\int_0^{400\pi} \sqrt{1-\cos 2x}dx$ is
(a) $200\sqrt{2}$
(b) $400\sqrt{2}$
(c) $800\sqrt{2}$
(d) None of these

50. $\int_0^2 |x^2+2x-3|dx$ is equal to

(a) 4
(b) 6
(c) 3
(d) 2